Solution

CET25M1 RELATIONS AND FUNCTIONS

Class 12 - Mathematics

1.

(c) mutually disjoint subsets

Explanation: An equivalence relation R gives a partitioning of the set A into mutually disjoint equivalence classes,i.e. union of equivalence classes is the set A itself.

2.

(c) transitive

Explanation: R = {(1, 2), (2, 3), (1, 3)} satisfying only the property of transitive relation.
i.e Its transitive i.e (a,b) $\in$ R and (b,c) $\in$ R $\rightarrow$ (a,c) $\in$ R $\forall$
a,b,c $\in$ A

3.

(c) a R b $\Leftrightarrow$ a < b

Explanation: a R b if a < b
Let a R a and a be an integer here.
Therefore, a < a, we can see that it is not possible.
It is not satisfying the condition, therefore, we can say that the given relation is not reflexive.
Now, we will check if the relation is symmetric or not.
Let a R b if a < b
We cannot write a < b as b < a i.e.,
$\Rightarrow$ a < b $\neq$ b < a
Hence, the given relation is not symmetric.
Now, we will check if the relation is transitive or not.
Let a R b and b R c if a < b and b < c is an even integer.
On adding both these, we get
$\Rightarrow$ a + b < b + c
On further simplification, we get
$\Rightarrow$ a < c
We can sway that the b R c
Therefore, the given relation is also transitive.
As the given relation is only transitive, therefore, the given relation is not an equivalence relation.

4.

(b) equivalence

Explanation: Given that,
R be a relation on T defined as aRb if a is congruent to b $\forall$ a, b $\in$ T
Now,
aRa $\Rightarrow$ a is congruent to a, which is true since every triangle is congruent to itself.
$\Rightarrow$ (a,a) $\in$ R $\forall$ a $\in$ T
$\Rightarrow$ R is reflexive.
Let aRb $\Rightarrow$ a is congruent to b
$\Rightarrow$ b is congruent to a
$\Rightarrow$ bRa
$\therefore$ (a,b) $\in$ R $\Rightarrow$ (b,a) $\in$ R $\forall$ a, b $\in$ T
$\Rightarrow$ R is symmetric.
Let aRb $\Rightarrow$ a is congruent to b and bRc $\Rightarrow$ b is congruent to c
$\Rightarrow$ a is congruent to c
$\Rightarrow$ aRc
$\therefore$ (a,b) $\in$ R and (b,c) $\in$ R $\Rightarrow$ (a,c) $\in$ R $\forall$ a, b,c $\in$ T

⇒ R is transitive.
Hence, R is an equivalence relation.

5.

(c) A

Explanation: $A_1,\ A_2, A_3, \ldots\ldots\ldots\ldots A_k$ be subsets of a set A such that $\bigcup_{i=1}^{n} A_i = A$ and $A_i \cap A_j = \phi$ for $i \neq j$

6.

(b) reflexive but neither symmetric nor transitive

Explanation: Let the given set be A = {1, 2, 3}
and R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)}
Reflexive Here, 1, 2, 3 ∈ A and (1, 1), (2, 2), (3, 3) ∈ R
i.e. for all a ∈ A, (a, a) ∈ R.
So, R is reflexive.
Symmetric Here, (1, 2) ∈ R but (2, 1) ∉ R.
So, R is not symmetric.
Transitive Here, (1, 2) ∈ R and (2, 3) ∈ R but (1, 3 } ∉ R.
So, R is not transitive.

7.

(b) injective only

Explanation: Let $x_1, x_2 \in N$

$f(x_1) = f(x_2)$

$\Rightarrow x_1^2 = x_2^2$
$\Rightarrow x_1^2 - x_2^2 = 0$
$\Rightarrow (x_1 + x_2)(x_1 - x_2) = 0$

$\Rightarrow x_1 = x_2$

$\{x_1 + x_2 \neq 0$ as $x_1, x^2 \in N\}$

Hence, f(x) is injective.
Also, the elements like 2 and 3 have no pre-image in N. Thus, f(x) is not surjective.

8.

(d) bijection

Explanation: Given that f : [-1/2, 1/2] → [π/2, π/2] where $f(x) = \sin^{-1}(3x - 4x^3)$

Put $x = \sin\theta$ in $f(x) = \sin^{-1}(3x - 4x^3)$
$\Rightarrow f(x = \sin\theta) = \sin^{-1}\left(3\sin\theta - 4\sin\theta^3\right)$

$\Rightarrow f(x) = \sin^{-1}(\sin 3\theta)$
$\Rightarrow f(x) = 3\theta$

$\Rightarrow f(x) = 3\sin^{-1}x$
If f(x) = f(y)
Then

$3\sin^{-1}x = 3\sin^{-1}y$
⇒ x = y
So, f is one-one.

$y = 3\sin^{-1}x$
$\Rightarrow x = \sin\frac{y}{3}$
∵ x ∈ R also y ∈ R so f is onto.
Hence, f is bijection.

9.

(b) an equivalence relation

Explanation: Check: (a, b)R (a, b) as
a + b = b + a
hence R is reflexive.

Now,let

(a, b) R (c, d) ,then,

a + d = b + c

⇒ c + b = d + a

⇒ (c, d) R (a, b)

=> R is symmetric

Now,

(a, b) R (c, d) and (c,d)R(e,f)Then,

a + d = b + c and

c + f = d + e

Adding,we get,

a + d + c + f = b + c + d + e

⇒ a + f = b + e

So (a, b) R (e, f)

R is transitive.

Hence R is an equivalence relation.

10. **(a)** neither one-one nor onto

Explanation: $f(x) = 2 + x^2$

For one-one, $f(x_1) = f(x_2)$

$\Rightarrow 2 + x_1^2 = 2 + x_2^2$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1 = \pm x_2$

$\Rightarrow x_1 = x_2$

or $x_1 = -x_2$

Thus, f(x) is not one-one.

For onto

Let f(x) = y such that y ∈ R

$\therefore x^2 = y - 2$

$\Rightarrow x = \pm\sqrt{y-2}$

Put y = -3, we get

$\Rightarrow x = \pm\sqrt{-3-2} = \pm\sqrt{-5}$

11.

(d) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Explanation: f(x) = $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ = $2\tan^{-1}x$

f(x) is one-one and onto

i.e., f'(x) > 0 or f'(x) < 0 and co-domain = range of f(x)

B = f(-1, 1) = $(2\tan^{-1}(-1), 2\tan^{-1}(1))$

$= \left(2 \times \left(-\frac{\pi}{4}\right), 2 \times \frac{\pi}{4}\right) = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

12. **(a)** many-one and into

Explanation: Given that f : R → R be given by $f(x) = [x]^2 + [x + 1] - 3$

As [x] is the greatest integer so for different values of x, we will get same value of f(x).

$[x]^2 + [x + 1]$ will always be an integer.

So, f is many-one.

Similarly, in this function co domain is mapped with at most one element of domain because for every x ∈ R, f(x) ∈ Z.

So, f is into.

13.

(d) transitive only

Explanation: According to the question:

R = {(a,b), (b,c), (a,c)}

Thus, R = {(b, c)} can only be transitive.

i.e (a,b) $\in$ R and (b, c) $\in$ R $\Rightarrow$ (a, c) $\in$ R

14. **(a)** one-one onto

Explanation: f(x) = $2x^3 + 2x^2 + 300x + 5 \sin x$

$f'(x) = 6x^2 + 4x + 300 + 5 \cos x$

$= 2(3x^2 + 2x + 147) + (6 + 5\cos x)$

Since $-1 \leq \cos x \leq 1 \ \forall x \in R$

$\therefore 6 + 5 \cos x > 0$

let $g(x) = 3x^2 + 2x + 147$

Since a = 3 > 0 and D = $(2)^2 - 4 \times 3 \times 147 < 0$

$\therefore g(x) > 0$

$f'(x) > 0$

$\therefore$ f(x) is an increasing function therefore said to be 1 -1 function.

Now, $f(\infty) = -\infty$ and $f(\infty) = \infty$

$\therefore$ f(x) is continuous also.

$\therefore$ Range of f = codomain of f = R

$\therefore$ f is onto.

15. **(a)** many-one and onto

Explanation: one-one: Let f(m) = f(n)

Three cases arise

i. m and n are both odd

clearly, f(m) = f(n) $\Rightarrow \frac{1}{2}(m+1) = \frac{1}{2}(m+1)$

= m + n

ii. m and n are both even

in this case f(m) = f(n) $\Rightarrow \frac{m}{2} + \frac{n}{2} \Rightarrow m = n$

iii. m is even n is odd

in this case f(m) = f(n)

$\Rightarrow \quad \frac{n}{2} = \frac{1}{2}(n+1)$

$\Rightarrow x \neq 3$

Hence f is not one-one.

Onto: let n $\in$ N (co-domai Now f(m)) = n

i.e. $m = \begin{cases} \frac{1}{2}(m+1), & \text{in is } odd \\ \frac{m}{2}, & m \text{ is even} \end{cases}$

So, for every n is co-domain. These exists m is domain r.t

n = f(m)

So, f is onto

Hence f is many-one onto function.

f: N $\rightarrow$ N: f(x) = $\begin{cases} \frac{1}{2}(n+1), \text{ when } n \text{ is odd} \\ \frac{n}{2}, \text{ when } n \text{ is even.} \end{cases}$

One-One function

When n is odd	When n is even
f(1) = 1	f(2) = 1
f(3) = 2	f(4) = 2

It is clear from the above that the function is many-one and Onto function

16.

(b) Transitive and symmetric

Explanation: The relation { }$\subset$ A $\times$ A on A is surely not reflexive. However, neither symmetry nor transitivity is contradicted. So { } is a transitive and symmetric relation on A.

17.

(b) many-one

Explanation: The given function f : R $\rightarrow$ R defined by f(x) = x^2 + x.

f(x) = X^2 + x
Now, for x = 0 and -1, we have
f(0) = 0 and f(-1) = 0
Hence, f(0) = f(-1) but 0 $\neq$ -1
$\Rightarrow$ f is not one one $\Rightarrow$ f is many one.

18.

(c) one-one and into
Explanation: We have, $f : R \rightarrow [0, \omega]$ defined by,
$f(x) = 10^x$
one - one: $f(x_1) = f(x_2)$
$\Rightarrow 10^{x_1} = 10^{x_2} \Rightarrow x_1 = x_2$
$\therefore$ f is one - one
onto: Here, $0 \in [0, \omega]$ (Co - domain)
Now, $f(x) = 0 \Rightarrow 10^x = 0$
But this is not possible for $x \in R$
$\therefore$ 0 does not has pre - image
$\therefore$ f is not onto

19.

(b) f is many one onto
Explanation: We have a function f : N $\rightarrow$ N, defined as
f(1) = F(2) = 1 and f(x) = x - 1, for every x > 2
For one-one: Since, f(1) = f(2) = 1 therefore 1 and 2 have same image, namely 1. So, f is not one-one. i.e. f is many one.
For onto: Note that y = 1 has two pre-images, namely 1 and 2. Now, let y $\in$ n, y $\neq$ 1, be any arbitrary element.
Then, y = f(x) $\Rightarrow$ y = x - 1
$\Rightarrow$ x = y + 1 > 2 for every y $\in$ N, y $\neq$ 1.
Thus, for every y $\in$ N, y $\neq$ 1, there exists
x = y + 1 such that
f(x) = f(y + 1) = y + 1 - 1 = y
Hence, f is onto.

20.

(d) one one and into

Explanation: f: R $\rightarrow$ R: f(x) = x^3
For One-One function
Let p, q be two arbitrary elements in R
then, f(p) = f(q)
$\Rightarrow$ P^3 = q^3
$\Rightarrow$ p = q
Thus, f(x) is one-one function.
For Onto function
Let v be an arbitrary element of R (co-domian)
Then, f(x) = v
x^3 = v
$\Rightarrow$ x = $\sqrt[3]{v}$
Since v $\in$ N
If v = 2, $\sqrt[3]{v}$ = 1.260, which is not possible as x $\in$ R
Thus, f(x) is not onto function. It is into function.

21.

(d) {(1, 1), (3, 3), (3, 1), (2, 3)}

Explanation:

i. R is reflexive if it contains ({1, 1), (2, 2) and (3, 3)}.
Since, (2, 2) $\in$ R. So, we need to add (1, 1) and (3, 3) to make R reflexive.

ii. R is symmetric if it contains {(2, 2), (1, 3), (3, 1), (3, 2), (2, 3)}.
Since, {(2, 2), (1, 3), (3,2)} $\in$ R. So, we need to add (3, 1) and (2, 3).
Thus, minimum ordered pairs which should be added in relation R to make it reflexive and symmetric are {(1, 1), (3, 3), (3, 1), (2, 3)}.

22.

(b) transitive but neither reflexive nor symmetric

Explanation: Since, x is greater than y $\forall$ x, y $\in$ N

Let (x, x) $\in$ R

For x R x, x > x is not true for any x $\in$ N.

Therefore, R is not reflexive.

Let (x, y) $\in$ R $\Rightarrow$ xRy

x > y

but y > x is not true for any x, y $\in$ N

Thus, R is not symmetric.

Let xRy and yRz

x > y and y > z

$\Rightarrow$ x > z

$\Rightarrow$ xRz

So, R is transitive.

23.

(d) $\left[\frac{1}{8}, \infty\right)$

Explanation: Here co-domain = $\left[0, \frac{\pi}{2}\right)$

For onto function, we have

Co-domain = Range = $0 \leq x < \frac{\pi}{2}$

This is valid if $x^2 + x + 2a \geq 0$ [$\because$ f(x) $\geq$ 0 i.e. $Ax^2 + Bx + C \geq 0$ then D $\leq$ 0 if A > 0]

i.e., $x^2 + x + 2a \geq 0 \Rightarrow 1^2 - 4 \times 1 \times 2a \leq 0$

$\Rightarrow$ 1 - 8a $\leq$ 0 $\Rightarrow 1 \leq 8a \Rightarrow 8a \geq 1 \Rightarrow a \geq \frac{1}{8}$

$\therefore a \in \left[\frac{1}{8}, \infty\right)$

24. **(a)** $A = (-\infty, 3]$ and $B = (-\infty, 1]$

Explanation: Given that f : A $\rightarrow$ B defined by f(x) = $-x^2 + 6x - 8$ is a bijection.

f(x) = $-x^2 + 6x - 8$

$\Rightarrow$ f(x) = $-(x^2 - 6x + 8)$

$\Rightarrow$ f(x) = $-(x^2 - 6x + 8 + 1 - 1)$

$\Rightarrow$ f(x) = $-(x^2 - 6x + 9 - 1)$

$\Rightarrow$ f(x) = $-[(x - 3)^2 - 1]$

Hence, $x \in (-\infty, 3]$ and $f(x) \in (-\infty, 1]$

25. **(a)** an equivalence relation

Explanation: an equivalence relation

Reflexivity: Let a $\in$ R

Then, $aa = a^2 > 0$

$\Rightarrow (a, a) \in R \forall a \in R$

So, S is reflexive on R.

Symmetry: Let $(a, b) \in S$

Then,

$(a, b) \in S$
$\Rightarrow ab \geq 0$
$\Rightarrow ba \geq 0$
$\Rightarrow (b, a) \in S \forall a, b \in R$
So, S is symmetric on R.
Transitive:
If $(a, b), (b, c) \in S$
$\Rightarrow ac \geq 0$ $[\because b2 \geq 0]$
$\Rightarrow (a, c) \in S$ for all a, b, c $\in$ set R
Hence,. S is an equivalence relation on R

26.

(c) both one-one and onto

Explanation: F(n) = { $\frac{n-1}{2}$, when n is odd; $\frac{-n}{2}$, when n is even
one - one : Let $n_1, n_2 \in N$
Case I: n_1 is even, n_2 is even
$\therefore f(n_1) - f(n_2) \Rightarrow \frac{-n_1}{2} = \frac{-n_2}{2} \Rightarrow n_1 = n_2$
Case II: n_1 is odd, n_2 is odd
$\therefore f(n_1) - f(n_2) \Rightarrow \frac{-n-1}{2} = \frac{-n-2}{2} \Rightarrow n_1 = n_2$
Case III: n_1 is even, n_2 is odd
$\therefore f(n_1) - \frac{-n_1}{2} =$ even as n_1 is even
$\therefore f(n_2)$ is odd, $\frac{n_2-1}{2}$ = even as n_2 is odd
But $f(n_1)$ takes values, -1, -2, -3,..... $f(n^2)$ take values, 0, 1, 2, 3,.....
$\therefore n_1 \neq n_2 \Rightarrow f(n_1) \neq f(n_2)$
Similarly n, is odd, n_2, is even, then $\therefore n_1 \neq n_2 \Rightarrow f(n_1) \neq f(n_2)$
$\Rightarrow$ f is one - one
onto: f(n) = { $\frac{n-1}{2}$, when n is odd; $\frac{-n}{2}$, when n is even}
$\therefore$ f(1) = 0, f(2) = 1, f(5) = 2, f(7) = 3, f(9) - 4, ... f(2)= -1, f(4) = -2, f(6) = -3, f(8) = -4, ...
$\therefore$ Range of f = {......, -2, -1, 0, 1, 2, 3, ...) = z
$\therefore$ F is onto

27. **(a)** symmetric

Explanation: A relation R on a non empty set A is said to be symmetric if xRy $\Leftrightarrow$ yRx, for all x,y $\in$ R. Clearly, $x^2 + y^2 = 1$ is same as $y^2 + x^2 = 1$ for all x,y $\in$R. Therefore, R is symmetric.

28.

(d) neither one-one nor onto

Explanation: Given that f : R $\rightarrow$ R be a function where
$f(x) = \frac{x^2-8}{x^2+2}$
Here, we can see that for negative as well as positive x we will get same value.
So, it is not one-one.
y = f(x)
$\Rightarrow \mathrm{y} = \frac{\mathrm{x}^2-8}{\mathrm{x}^2+2}$
$\Rightarrow y(x^2 + 2) = (x^2 - 8)$
$\Rightarrow x^2(y - 1) = -2y - 8$
$\Rightarrow \mathrm{x} = \sqrt{\frac{2\mathrm{y}+8}{1-\mathrm{y}}}$
For y = 1 , no x is defined.
So, f is not onto.

29. **(a)** 3

Explanation: For R to be reflexive, (b, b) and (c, c) should belong to R and for R to be transitive [a, c) should belong to R, as (a, b) ∈ R and (b, c) ∈ R. Hence, minimum number of ordered pairs to be added in R is 3.

30. **(a)** not symmetric

Explanation: Here, R is not reflexive as x is not 7 cm taller than x.

R is not symmetric as if x is exactly 7 cm taller than y, then y cannot be 7 cm taller than x and R is not transitive as if x is exactly 7 cm taller than y and y is exactly 7 cm taller than z, then x is exactly 14 cm taller than z.

31.

(b) None of these

Explanation: Number of onto function

$= 3^6 - {}^3C_1 (3 - 1)^6 + {}^3C_2(3 - 2)^6 - {}^3C_3 (3 - 3)^6$

= 36 - 3 × 26 + 3 × 1 = 3^6 - 3 × 26 + 3

= 3 × (35 - 26 + 1) = 3(243 - 64 + 1)

= 3 × (244 - 64) = 3 × 180 = 540

32.

(d) m ≥ n

Explanation: Given, R has m ordered pairs.

Since R is reflexive relation on A, therefore (a, a) ∈ R for all a ∈ A.

The minimum number of ordered pairs in R is n.

Therefore, m ≥ n.

33. **(a)** Symmetric but neither reflexive nor transitive

Explanation: We observe the following properties of S.

Reflexivity:

Let a be an arbitrary element of R. Then,

$a \in R$

$\Rightarrow a^2 + a^2 \neq 1 \forall a \in R$

$\Rightarrow (a, a) \notin S$

So, S is not reflexive on R.

Symmetry: Let (a, b) ∈ R

$\Rightarrow a^2 + b^2 = 1$

$\Rightarrow b^2 + a^2 = 1$

⇒ (b, a) ∈ S for all a, b ∈ R

So, S is symmetric on R.

Transitivity:

Let (a, b) and (b, c) ∈ S

$\Rightarrow a^2 + b^2 = 1$ and $b^2 + c^2 = 1$

Adding the above two, we get

$a^2 + c^2 = 2 - 2b^2 \neq$ for all a, b, c, ∈ R

So, S is not transitive on R.

Hence, S is not an equivalence relation on R.

34.

(d) one – one onto

Explanation: Injectivity: Let $x_1, x_2 \in R$ such that $f(x_1) = f(x_2)$. Then, $f(x_1) = f(x_2) \Rightarrow 3x_1 = 3x_2 \Rightarrow x_1 = x_2$. So, f : R → R is one –one.

Surjectivity: Let y ∈ R, Then $f(x) = y \Rightarrow 3x = y \Rightarrow x = \frac{y}{3}$, Clearly, $\frac{y}{3} \in R \; for \; any \; y \in R$ such that $f\left(\frac{y}{3}\right) = 3\left(\frac{y}{3}\right) = y$. So, Let f : $R \rightarrow R$ is onto.

35.

(d) Non-symmetric

Explanation: Non-symmetric

36.

(d) symmetric but neither reflexive nor transitive

Explanation: Since the line L is not perpendicular to itself so the relation is not reflexive.

If the line L is perpendicular to line M then the line M is also perpendicular to line L to the relation is symmetric.
If line L is perpendicular to line M and M is perpendicular to line F then Line L and Line F are parallel to each other so the relation is not transitive.

37. **(a)** onto but not one-one

Explanation: Given function f : Z $\rightarrow$ Z defined as

$f(x)=\begin{Bmatrix} \frac{x}{2}, \text{ when } x \text{ is even} \\ 0, \text{ when } x \text{ is odd} \end{Bmatrix}$

For x = 3, f(x) = 0
For x = 5, f(x) = 0
But 3 ≠ 5
So, f is not one-one.
y = f(x)
$\because$ x $\in$ R $\Rightarrow$ y $\in$ R
$\therefore$ Codomain= Range
Hence, f is not one-one but onto.

38.

(d) Equivalence
Explanation: Equivalence

39.

(d) reflexive, transitive but not symmetric
Explanation: S: a S b $\Leftrightarrow$ a $\geq$ b
Since a = a ∀a ∈ R, therefore a $\geq$ a always. Hence (a, a) always belongs to S $\forall$ a $\in$ R. Therefore, S is reflexive.
If a $\geq$ b then b ≤ a $\neq$ b ≥ a. Hence if (a, b) belongs to S, then (b, a) does not always belongs to S. Hence S is not symmetric.
If a $\geq$ b and b $\geq$ c, therefore a $\geq$ c. Hence if (a, b) and (b, c) belongs to S, then (a, c) will belong to S $\forall$ a, b, c $\in$ R. Hence, S is transitive.

40.

(c) one-one
Explanation: Given that, A = {1, 2, 3} and B = {4, 5, 6, 7}
Now, f : A $\rightarrow$ B is defined as
f = {(1, 4), (2, 5), (3, 6)}
Therefore, f(1) = 4, f(2) = 5, f(3) = 6
It can be seen that the images of distinct elements of A under f are distinct.
Hence, function f is one-one. But, f is not onto, as 7 $\in$ B does not have a preimage in A.

41.

(b) neither one-one nor onto
Explanation: Since, greatest integer function, give integer values
Range (f) = Integers $\neq$ R, so it is not onto.
and [2 $\cdot$ 3] = [2 $\cdot$ 4] = 2
$\Rightarrow$ f is not one-one.

42.

(b) neither injective nor surjective
Explanation: Since, f(n) = $\begin{cases} n^2, & \text{for } n \text{ odd} \\ 2n+1, & \text{for } n \text{ even} \end{cases}$

$\therefore$ f(1) = 1^2 = 1,
f(2) = 2(2) + 1 = 5
f(3) = 3^2 = 9,
f(4) = 2(4) + 1 = 9
$\because$ f(3) = f(4)
So, f is not injective.
Also, f is not surjective as some element of N (codomain) is not the image of any element of N.

43.

(c) 1

Explanation: This is because relation R is reflexive as (1, 1), (2, 2), (3, 3) $\in$ R.

Relation R is symmetric as (1, 2), (2, 1) $\in$ R and (1, 3), (3, 1) $\in$ R.

But relation R is not transitive as (3, 1), (1, 2) $\in$ R but (3, 2) R.

Now, if we add any one of the two pairs (3, 2) and (2, 3) (or both) to relation R,

Then, the relation R will become transitive.

Therefore, the total number of desired relations is one.

44.

(c) $\left[0, \frac{\pi}{2}\right]$ and [-1, 1]

Explanation: Since, f : X $\rightarrow$ Y and f(x) = sin x

Now, take option $\left[0, \frac{\pi}{2}\right]$ and [-1, 1].

Domain = $\left[0, \frac{\pi}{2}\right]$

Range = [-1,1]

For every value of x, we get unique value of y.

But the value of y in [-1, 0) does not have any pre image.

$\therefore$ Function is one-one but not onto.

45.

(c) reflexive and symmetric

Explanation: According to the condition,

(x,y)$\in$R $\implies$ |x – y| $\leq$ 1

Reflexive: let (x,x) $\in$R $\implies$ |x-x|=0<1

$\Rightarrow$R is Reflexive

Symmetric:

If (x,y) $\in$ R $\implies$ |x-y|$\leq$1

and (y,x) $\in$ R $\implies$ |y-x|$\leq$1 [Since |x-y|=|y-x|]

$\Rightarrow$R is Symmetric

Transitive:

If (x,y)$\in$R$\Rightarrow$|x-y|$\leq$1

and (y,z)$\in$R$\Rightarrow$|y-z|$\leq$1

$\Rightarrow$|x-y|=|x-y+y-z|

$\leq$|x-y|+|y-z|$\leq$1+1=2

$\Rightarrow$|x-z|$\leq$2

$\therefore$R is not transitive

46. **(a)** 5

Explanation: Let n be the number of elements in the set A , then the maximum number of equivalence relations defined on A are given by : 2n – 1. Here A={1,2,3} $\Rightarrow$ n(A)=3, then the number of relations of A is given by 2 x 3 -1 = 5.

47.

(b) neither one-one nor onto

Explanation: Given, a function f : R $\rightarrow$ R defined as f(x) = x^2.

For one-one: Here, at x = 1, f(1) = 1

and at x = -1, f(-1) = $(-1)^2$ = 1

Thus, f(1) = f(-1) = 1, but 1 $\neq$ -1

So, f is not one-one.

For onto: Let y $\in$ R (codomain) be any arbitrary element.

Then, y = f(x) $\Rightarrow$ y = x^2 $\Rightarrow$ x = $\pm\sqrt{y}$

Now, for y = -2 $\in$ R, x = $\pm\sqrt{-2} \notin R$

So, f is not onto.

Hence, given function is neither one-one nor onto.

48. **(a)** an equivalence relation

Explanation: For R to be reflexive :

R = { (a,b) : a - b is divisible by 3 }
a - a = 0 , 0 is divisible by all numbers , so it is divisible by 3 therefore (a,a) belongs to R
Hence, R is reflexive relation.
For R to be symmetric :
R={ (a, b) : a - b is divisible by 3 }
Let (a, b) belongs to R, then a - b is divisible by 3.
=> b - a is divisible by 3
=> (b, a) also belongs to R
Thus, R is a symmetric relation.
For R to be transitive,
let a,b, c be such that (a,b) belongs to R, (b,c) belongs to R.Then,
a-b is divisible by 3
=> a-b=3k, where k is an integer.
Also, b-c is divisible by 3
=> b-c=3p, where p is an integer.
Adding the above two equations, we get
a-c = 3(k-p) = 3q, q is also an integer
therefore (a,c) also belongs to R
therefore R is transitive relation.
As relation R is reflexive, symmetric, transitive
Therefore R is an equivalence relation.

49.

(d) {1, 3}, {2, 4, 5}, {6}
Explanation: Conditions for the partition sub-sets to be an equivalence relation:
The partition sub-sets must be disjoint i.e.there is no common elements between them
Their union must be equal to the main set (super-set)
Here, the set A={1,2,3,4,5,6},the partition sub-sets {1,3},{2,4,5},{6} are pairwise disjoint and their union i.e. {1,3} U {2,4,5} U {6} = {1,2,3,4,5,6} = A, which is the condition for the partition sub-sets to be an equivalence relation of the set A.

50.

(c) f is both one – one and onto
Explanation: A function f: X → Y is defined to be one – one (or injective), if f$x_1 \neq x_2$ in X ⇒ $f(x_1) \neq f(x_2)$ in Y. and $R_f = Y$.

51.

(d) Reflexive and transitive but not symmetric
Explanation: The relation R is not symmetric, (1,2) ∈ R , but (2,1) ∉ R , (1,3) ∈R ,but (3,1) ∉ R , (3,2) ∈ R, but (2,3) ∉ R.

52. **(a)** Reflexive, transitive but not symmetric
Explanation: Given, R = {(x, y) : y is divisible by x]
and A = {1, 2, 3, 4, 5, 6}
Reflexive: Let x ∈ A be any arbitrary element.
We know that, x is divisible by x.
[∵ every real number except zero is divisible by itself]
⇒ (x, x) ∈ R
Since, x ∈ arbitrary element, therefore (x, x) ∈ R, ∀ x ∈ A. So, R is reflexive.
Symmetric: Clearly, 2, 4 ∈ A and 4 is divisible by 2, but 2 is not divisible by 4.
∴ (2, 4) ∈ R but (4, 2) ∉ R
So, R is not symmetric.
Transitive: Let x, y, z ∈ A such that (x, y) ∈ R and (y, z) ∈ R.
Now, as (x, y) ∈ R, therefore y is divisible by x.
i.e. $\frac{y}{x}$ = k_1 (say) ...(i)
where, k_1 is a natural number
and as (y, z) ∈ R, therefore z is divisible by y.
i.e. $\frac{z}{y}$ = K_2 (say) ...(ii)

where, k_2 is a natural number.

On multiplying Eqs. (i) and (ii), we get

$\frac{y}{x} \times \frac{z}{y} = k_1 k_2 \Rightarrow \frac{z}{x} = k_1 k_2$

where, k_1k_2 is a natural number.

$\therefore$ z is divisible by x.

Thus, (x, z) $\in$ R, for {x, y), (y, z) $\in$ R,

i.e. (x, y) $\in$ R, (y, z) $\in$ R $\Rightarrow$ (x, z) $\in$ R

Hence, R is transitive.

53. **(a)** an equivalence relation

Explanation: Given Relation R = {(1, 1), (2, 2), (3, 3)}

Reflexive: If a relation has {(a, b)} as its element, then it should also have {(a, a), (b, b)} as its elements too.

Symmetric: If a relation has (a, b) as its element, then it should also have {(b, a)} as its element too.

Transitive: If a relation has {(a, b), (b, c)} as its elements, then it should also have {(a,c)} as its element too.

Now, the given relation satisfies all these three properties.

Therefore, its an equivalence relation.

54.

(b) 2

Explanation: Total possible pairs {(1, 1) (1, 3), (1, 5), (3, 3), (3, 1), (3, 5) (5, 5), (5, 1), (5, 3)}

1st equivalence relation

R_1 = {(1, 1,), (5, 5), (3, 3), (1, 3), (3, 1)}

2nd equivalence relation

R_2 = {(1, 1,), (5, 5), (3, 3), (1, 3), (3, 1), (3, 5), (5, 3)}

$\therefore$ no of possible equivalence relation

= 2

55.

(c) reflexive but not symmetric

Explanation: As (1, 1), (2, 2), (3, 3) $\in$ R, therefore R is reflexive. Since (1,2) $\in$ R, but (2,1) $\notin$ R. Therefore,R is not symmetric.

56.

(d) one-one and onto

Explanation: one-one and onto

57.

(d) Symmetric but neither reflexive nor transitive.

Explanation: The relation R is symmetric only , because if L_1 is perpendicular to L_2 ,then L_2 is also perpendicular to L_1,but no other cases that is reflexive and transitive is not possible.

58. **(a)** reflexive, symmetric and transitive

Explanation: By definition of Equivalence Relation, a relation is said to be equivalence if it is reflexive, symmetric and transitive.

59.

(d) Equivalence relation

Explanation: Equivalence relation

60.

(c) $\left[\frac{1}{4}, \infty\right)$

Explanation: From definition of onto function,

Range of function = Codomain of function = $\left[0, \frac{\pi}{2}\right)$

$\Rightarrow 0 \leq \tan^{-1}\left(x^2 + x + a\right) < \frac{\pi}{2}$

$\Rightarrow 0 \leq (x^2 + x + a) < \infty$

$\Rightarrow x^2 + x + a > 0 \ \forall x \in R$

Hence D ≤ 0
$\Rightarrow 1^2 - 4a \leq 0$
$\Rightarrow 4a \geq 1$
$\Rightarrow a \geq \frac{1}{4}$
$\Rightarrow a \in \left[\frac{1}{4}, \infty\right)$

61. **(a)** not anti symmetric
Explanation: A relation R on a non empty set A is said to be reflexive if xRx for all x $\in$ R , Therefore , R is not reflexive.
A relation R on a non empty set A is said to be symmetric if xRy $\Leftrightarrow$ yRx, for all x , y $\in$ R. Therefore, R is not symmetric.
A relation R on a non empty set A is said to be antisymmetric if xRy and yRx $\Rightarrow$ x = y , for all x , y $\in$ R. Therefore, R is not antisymmetric.

62.
(b) one-one and onto
Explanation: Here f: A $\times$ B $\rightarrow$ B $\times$ A Such that
f(x, b) = (b, a)
One-One: - Let f(a_1, b_1) = f(a_2, b_2)
$\Rightarrow$ (b_1, a_1) = (b_2, a_2)
$\Rightarrow$ a_1 = a_2 and b_1 = b_2
$\Rightarrow$ (a_1, b_1) = (a_2, b_2)
$\therefore$ f is one - one.
Onto: - Let (x, y) $\in$ B $\times$ A then
(x, y) = f(a, b)
$\Rightarrow$ (x, y) = (b, a)
$\Rightarrow$ x = b and y = a
So, for every (b, a) $\in$ B $\times$ A
There exists (a, b) $\in$ A $\times$ B
$\Rightarrow$ f is onto
Hence f is a Bijection.

63.
(d) an equivalence relation
Explanation: R is reflexive since every triangle is congruent to itself.
Further, $(T_1, T_2) \in R$
$\Rightarrow$ T_1 is congruent to T_2
$\Rightarrow$ T_2 is congruent to $T_1 \Rightarrow (T_2, T_1) \in R$.
Hence, R is symmetric.
Moreover, (T_1, T_2), (T_2, T_3) $\in$ R
$\Rightarrow$ T_1 is congruent to T_2 and T_2 is congruent to T_3
$\Rightarrow$ T_1 is congruent to T_3
$\Rightarrow (T_1, T_3) \in R$.
Therefore, R is an equivalence relation.

64.
(c) Transitive
Explanation: Let R be a relation on the set of all intergers Z, defined by aRb $\Leftrightarrow$ b $\forall$ a, b $\in$ Z

i. **Reflexive:** For 1 $\in$ Z
1 $\not R$ 1 as 1 $\not>$ 1 so (1,1) $\notin$ R $\Rightarrow$ R is not reflexive on Z.
ii. **Symmetric:** (3, 2) e R as 3 > 2
But (2, 3) $\notin$ R as 2 $\not>$ 3
Hence R is not symmetric on Z.
iii. **Transitive:** Let (a, b) $\in$ R and (b, c) $\in$ R a > b and b > c
Now a > b > c $\Rightarrow$ a > c $\Rightarrow$ (a, c) $\in$ R
Hence R is a transitive relation on Z.

65.

(d) transitive only

Explanation: 1 belongs to A but (1,1) does not belong to R,so R is not reflexive .(2,3) is in R but (3,2) is not in R,so R is not transitive.since R consists of only one element (2,3),so it is transitive

66. **(a)** transitive but neither reflexive nor symmetric

Explanation: We have,
R = {(x, y) : x + 4y = 10, x, y ∈ N}
R = {(2, 2), (6, 1)}
Here, (1, 1), (3, 3), .., ∉ R
Thus, R is not reflexive.
(6,1) ∈ R but (1, 6) ∉ R
Hence, R is not symmetric.
Let (x, y) ∈ R ⇒ x + 4y = 10 and (y, z) ∈ R
y + 4z = 10 ⇒ (x, z) ∈ R
So, R is transitive.

67.

(c) f(x) is neither one-one nor onto

Explanation: f(x) is neither one-one nor onto

68. **(a)** An equivalence relation

Explanation: R = {(P, Q): distance of point P from the origin is the same as the distance of point Q from the origin}
Let origin be O
Hence OP = OQ
So,
R = {(P, Q): OP = OQ}

Check reflexive

since, point P and point P are same
Distance of Point P from origin $=$ Distance of point P from the origin
So, (P, P) ∈ R,
$\therefore$ R is reflexive.

Check Symmetric

If Distance of Point P from origin = Distance of point Q from the origin
then,
Distance of Point Q from origin = Distance of point P from the origin
So, if (P, Q) ∈ R, then (Q, P) ∈ R
$\therefore$ R is symmetric.

Check transitive

If Distance of Point P from origin = Distance of point Q from the origin
& Distance of Point Q from origin = Distance of point 5 from the origin
then
Distance of Point P from origin = Distance of point S from the origin
So, if (P, Q) ∈ R & if (Q, S) ∈ R, then (P, S) ∈ R
$\therefore$ R is transitive.
Since R is reflexive, symmetric & transitive.
$\therefore$ R is an equivalence relation.

69.

(c) f is not defined

Explanation: Because , $\frac{1}{x}\ \ is\, not\, defined\, for\, x\ = 0,\ as\, 0 \in R,\ \therefore f\, is\, not\, defined.$

70.

(d) all the three options

Explanation: R is reflexive,since every element is related to itself.
Also, R is symmetric as there is no element of the form (x,y) in R,so we don't have to show anything.It is vacuously true.

By the same reason as in symmetry of R,R is transitive ad it no elements of the form (x,y).This,R is an equivalnce relationship.

71.

(d) injective

Explanation: Since 1,2,3,4,5 have no pre-image hence it is not surjective so it is only injective.

72.

(b) mutually disjoint subsets

Explanation: An equivalence relation R gives a partitioning of the set A into mutually disjoint equivalence classes,i.e. union of equivalence classes is the set A itself. Any two equivalence classes i.e. subsets are either equal or disjoint.

73.

(d) symmetric

Explanation: Let $(x,y) \in R$, such that $x \perp y$.

We can also write from above that, $y \perp x$.

Hence, $(y,x) \in R$

So, They are symmetric.

74.

(b) an equivalence relation

Explanation: For R = {(a, b): a+b is even,$\forall$ a,b $\in$ N},

Reflexive property: (a, b) $\in$ R $\Rightarrow$ a + b is even. Now put b = a, a + a is also even $\Rightarrow$(a, a) $\in$ R, R is reflexive.

Symmetric property: (a,b) $\in$ R $\Rightarrow$ a + b is even $\Rightarrow$ b + a is also even $\Rightarrow$ (b, a) $\in$ R $\Rightarrow$ R is Symmetric.

Transitive property: (a, b) $\in$ R,(b,c) $\in$ R $\Rightarrow$ a + b is even and b + c is even $\Rightarrow$ a + c is also even $\Rightarrow$ (a, c) $\in$ R, so R is transitive. Hence R is an equivalence relation.

75.

(d) many-one and into

Explanation: f: C $\to$ R: f(z) = |z|

One-One function

Let p, q be two arbitrary elements in R+

then, f(p) = f(q)

$\Rightarrow$ |p| = |q|

$\Rightarrow$ p = q or -q

Thus f(x) is many-one function.

Onto function

f(x) can only assume values between 0 and ∞, which is not equal to the codomain, which is R.

thus, f(x) is not onto function. It is into.