Solution

CET25M2 INVERSE TRIGONOMETRIC FUNCTIONS

Class 12 - Mathematics

1.
(b) $\frac{2\pi}{3}$
Explanation: Let the principle value be given by x
Now, let x = $\cos^{-1}\left(\frac{-1}{2}\right)$
$\Rightarrow \cos x = \frac{-1}{2}$
$\Rightarrow$ cos x = -cos $\left(\frac{\pi}{3}\right)\left(\because \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}\right)$
$\Rightarrow$ cos x = cos $\left(\pi - \frac{\pi}{3}\right)(\because -\cos(\theta) = \cos(\pi - \theta))$
$\Rightarrow$ x = $\frac{2\pi}{3}$

2. **(a)** 0
Explanation: 2 $\cos^{-1}\left(\frac{-1}{2}\right)$ + 2 $\sin^{-1}\left(\frac{-1}{2}\right)$ - $\cos^{-1}(-1)$
= 2(π - $\cos^{-1}\left(\frac{1}{2}\right)$) - 2 $\sin^{-1}\left(\frac{1}{2}\right)$ - (π - $\cos^{-1}$ 1)
= 2(π - $\frac{\pi}{3}$ - 2$\left(\frac{\pi}{6}\right)$ - (π - 0)
$= 2\left(\frac{2\pi}{3}\right) - \frac{\pi}{3} - \pi$
$= \frac{4\pi}{3} - \frac{\pi}{3} - \pi = \frac{4\pi - \pi - 3\pi}{3}$ = 0

3.
(c) $-\frac{\pi}{3}$
Explanation: Let us take
$\tan^{-1}(\sqrt{3}) = x$ Then we get,
$tanx = \sqrt{3} = tan\frac{\pi}{3}$
$\Rightarrow\ x = \frac{\pi}{3}$
We know that range of the principle value branch of $\tan^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
Therefore,x = $\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$
Let $\sec^{-1}(-2) = y$ then we get,
$\sec y = -2 = -\sec\frac{\pi}{3} = \sec\left(\pi - \frac{\pi}{3}\right) = \sec\left(\frac{2\pi}{3}\right)$
We know that range of the principle value branch of $\sec^{-1}$ is $[0, \pi]$
Therefore, $\sec^{-1}(-2) = \frac{2\pi}{3}$
Hence, $\tan^{-1}\sqrt{3} -\ \sec^{-1}(-2) = \frac{\pi}{3} - \frac{2\pi}{3} = -\frac{\pi}{3}$

4.
(d) $-1 \leq x \leq 1$
Explanation: $sin^{-1}x$ is defined only for [-1,1]
$\therefore$ Domain of f(x) = $\sin^{-1}$x is [-1, 1].

5. **(a)** $\frac{\pi}{4}$
Explanation: $sin^{-1}\left(\frac{1}{\sqrt{5}}\right) + cot^{-1}(3)$
$\Rightarrow \tan^{-1}\left(\frac{1}{2}\right) + tan^{-1}\left(\frac{1}{3}\right)$ because $\frac{1}{2}.\frac{1}{3} < 1$
$\Rightarrow \tan^{-1}\left(\frac{\left(\frac{1}{2}\right)+\left(\frac{1}{3}\right)}{1-\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)}\right)$
$\Rightarrow \tan^{-1}(1) = \frac{\pi}{4}$

6. **(a)** $\frac{\pi}{4}$
Explanation: $\sin^{-1}(\sin\frac{3\pi}{4}\ \ ) \ = \sin^{-1}(\sin\left(\pi - \frac{\pi}{4}\right)$
$= \sin^{-1}(\sin\frac{\pi}{4}) = \frac{\pi}{4}$.

7.

(d) [–1, 1]

Explanation: $y = \sin^{-1}(-x^2) \Rightarrow \sin y = -x^2$

i.e. $-1 \leq -x^2 \leq 1$ (since $-1 \leq \sin y \leq 1$)

$\Rightarrow 1 \geq x^2 \geq -1$

$\Rightarrow 0 \leq x^2 \leq 1$

$\Rightarrow |x| \leq 1$ i.e. $-1 \leq x \leq 1$

8.

(b) $\frac{-\pi}{10}$

Explanation: $\sin^{-1}\left(\cos \frac{3\pi}{5}\right)$

$= \sin^{-1} \sin\left(\frac{\pi}{2} - \frac{3\pi}{5}\right)$

$= \sin^{-1} \sin\left(\frac{5\pi-6\pi}{10}\right)$

$= \sin^{-1} \sin\left(\frac{-\pi}{10}\right)$

$= \frac{-\pi}{10}$

9.

(b) π

Explanation: We have,

$\tan^{-1}(\sqrt{3}) + \cos^{-1}\left(-\frac{1}{2}\right) = \tan^{-1}(\tan \frac{\pi}{3}) + \cos^{-1}(-\cos \frac{\pi}{3})$

$= \frac{\pi}{3} + \cos^{-1} |\cos(\pi - \frac{\pi}{3})| = \frac{\pi}{3} + \cos^{-1} (\cos \frac{2\pi}{3})$

$= \frac{\pi}{3} + \frac{2\pi}{3} = \frac{3\pi}{3} = \pi$

10.

(d) 0.96

Explanation: $\sin(2\tan^{-1}(0.75))$

Let, $\tan^{-1}(0.75) = \theta$

$\Rightarrow \tan^{-1}\left(\frac{3}{4}\right) = \theta$

$\Rightarrow \tan\theta = \frac{3}{4}$

As, $\tan\theta = \frac{3}{4}$, so

$\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$ ....(1)

Now,

$\sin(2\tan^{-1}(0.75)) = \sin 2\theta$

$= 2 \sin\theta \cos\theta$

$= 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)$

$= \frac{24}{25}$

So, $\sin(2\tan^{-1}(0.75)) = 0.96$

11. **(a)** $\frac{-\pi}{4}$

Explanation: Let the principle value be given by x

also, let $x = \operatorname{cosec}^{-1}(-\sqrt{2})$

$\Rightarrow \operatorname{cosec} x = -\sqrt{2}$

$\Rightarrow \operatorname{cosec} x = -\operatorname{cosec}\left(\frac{\pi}{4}\right) \left(\because cosec\left(\frac{\pi}{4}\right) = \sqrt{2}\right)$

$\Rightarrow \operatorname{cosec} x = \operatorname{cosec}\left(-\frac{\pi}{4}\right)$ ($\because -\operatorname{cosec}(\theta) = \operatorname{cosec}(-\theta)$)

$\Rightarrow x = -\frac{\pi}{4}$

12. **(a)** $\theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Explanation: As $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ so $-1 \leq \sin\theta \leq 1$ but we have $0 < \sin\theta < 1$

$\log_{\sin\theta}(\cos^2\theta - \sin^2\theta) = 2$

$\Rightarrow \cos^2\theta - \sin^2\theta = (\sin\theta)^2$ and $0 < \sin\theta < 1$

$\Rightarrow \cos^2\theta = 2\sin^2\theta$ where $\sin\theta \in (0, 1)$

$\therefore \tan^2\theta = \frac{1}{2} \Rightarrow \tan\theta = \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$

Also $\cos^2\theta - \sin^2\theta = \sin^2\theta$, $\sin\theta \in (0, 1)$

$\Rightarrow 1 - 2\sin^2\theta = \sin^2\theta$, $\sin\theta \in (0, 1)$

$\Rightarrow 1 = 3\sin^2\theta$ where $\sin\theta \in (0, 1)$

$\Rightarrow \sin^2\theta = \frac{1}{3}$, $\sin\theta \in (0, 1)$

$\Rightarrow \sin\theta = \pm\frac{1}{\sqrt{3}}$, $\sin\theta \in (0, 1)$

But $\sin\theta = -\frac{1}{\sqrt{3}}$ is not possible, as base of log is +ve and not equal to 1.

$\therefore \sin\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$.

13.

(b) $x \in [0, \pi]$

Explanation: $\cos^{-1}(\cos x) = x$ if $0 \leqslant x \leqslant \pi$ i.e. if , $x \in [0, \pi]$

14.

(d) $-\frac{\pi}{3}$

Explanation: $\sin^{-1}\sin(600)^o = \sin^{-1}[\sin(540^o + 60^o)]$

$= \sin^{-1}[\sin(3\pi + \frac{\pi}{3})]$

$= \sin^{-1}[-\sin\frac{\pi}{3}]$

$= \sin^{-1}[\sin\left(-\frac{\pi}{3}\right)]$

$= -\frac{\pi}{3}$

15.

(d) $\frac{\pi}{3}$

Explanation: $\cos^{-1}\left(\cos\left(-\frac{\pi}{3}\right)\right) = \cos^{-1}(\cos\frac{\pi}{3}) = \frac{\pi}{3}$, because, $\cos\theta$ is positive in fourth quadrant.

16. **(a)** $\frac{\sqrt{x^2-1}}{x}$

Explanation: if $\theta = \cos^{-1}\left(\frac{1}{x}\right)$

$\Rightarrow \cos\theta = \frac{1}{x} = \frac{Base}{Hyp.} \Rightarrow \tan\theta = \frac{Perp.}{Base} = \frac{\sqrt{x^2-1}}{x}$

17. **(a)** $\frac{5\pi}{6}$

Explanation: Let the principle value be given by x

also, let $x = \sec^{-1}\left(\frac{-2}{\sqrt{3}}\right)$

$\Rightarrow \sec x = \frac{-2}{\sqrt{3}}$

$\Rightarrow \sec x = -\sec\left(\frac{\pi}{6}\right)\left(\because \sec\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}}\right)$

$\Rightarrow \sec x = \sec\left(\pi - \frac{\pi}{6}\right)$ $(\because -\sec(\theta) = \sec(\pi - \theta))$

$\Rightarrow x = \frac{5\pi}{6}$

18.

(d) ϕ

Explanation: Since, domain of $\sec^{-1} x$ is R - (-1, 1).

$\Rightarrow (-\infty, -1] \cup [1, \infty)$

So, there is no set of values exist for $\sec^{-1}\frac{1}{2}$.

So, ϕ is the answer.

19.

(c) [0, 1]

Explanation: We have $f(x) = \cos^{-1}(2x - 1)$

Since, $-1 \leq 2x - 1 \leq 1$

$\Rightarrow 0 \leq 2x \leq 2$

$\Rightarrow 0 \leq x \leq 1$

$\therefore x \in [0,1]$

20.

(d) $\frac{\pi}{2}$

Explanation: Let $\cos^{-1}(-1) = A \Rightarrow \cos A = -1 \Rightarrow \cos A = \cos\pi \therefore A = \pi$

and $\sin^{-1}(1) = B \Rightarrow \sin B = 1 \Rightarrow \sin B = \sin\left(\frac{\pi}{2}\right)$

$\therefore B = \left(\frac{\pi}{2}\right)$

$\therefore \cos^{-1}(-1) - \sin^{-1}(1) = \pi - \frac{\pi}{2} = \frac{\pi}{2}$ .Which is the required solution.

21.

(c) $\frac{7}{24}$

Explanation: We have to find,

cot ($\cos^{-1}$) $\frac{7}{25}$

Let, $\cos^{-1}$ ($\frac{7}{25}$) = A

$\Rightarrow \cos A = \frac{7}{25}$

Also,cot A = cot ($\cos^{-1}$($\frac{7}{25}$))

As, sin A = $\sqrt{1 - \cos^2 A}$

So, sin A = $\sqrt{1 - \left(\frac{7}{25}\right)^2}$

$\Rightarrow \sin A = \sqrt{1 - \frac{49}{625}}$

$\Rightarrow \sin A = \sqrt{\frac{625-49}{625}}$

$\Rightarrow \sin A = \sqrt{\frac{576}{625}}$

$\Rightarrow \sin A = \frac{24}{25}$

We need to find cot A

cot A = $\frac{\cos A}{\sin A}$

$\Rightarrow \cot A = \frac{\left(\frac{7}{25}\right)}{\left(\frac{24}{25}\right)}$

$\Rightarrow \cot A = \frac{7}{24}$

So, cot ($\cos^{-1}$ ($\frac{7}{25}$)) $= \frac{7}{24}$

22. **(a)** $\frac{-\pi}{6}$

Explanation: Let the principle value be given by x

also, let x = $\sin^{-1}\left(\frac{-1}{2}\right)$

$\Rightarrow \sin x = \frac{-1}{2}$

$\Rightarrow$ sin x = -sin $\left(\frac{\pi}{6}\right) \left(\because \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}\right)$

$\Rightarrow$ sin x = sin $\left(-\frac{\pi}{6}\right) (\because -\sin(\theta) = \sin(-\theta))$

$\Rightarrow$ x = $-\frac{\pi}{4}$

23.

(b) $\frac{\pi}{6}$

Explanation: Let the principle value be given by x

Now, let x = $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\Rightarrow \cos x = \frac{\sqrt{3}}{2}$

$\Rightarrow$ cos x = cos $\left(\frac{\pi}{6}\right) \left(\because \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}\right)$

$\Rightarrow$ x = $\frac{\pi}{6}$

24.

(d) $\frac{-\pi}{3}$

Explanation: Let the principle value be given by x

also, let x = tan^{-1} $(-\sqrt{3})$
$\Rightarrow$ tan x = $-\sqrt{3}$
$\Rightarrow$ tan x = -tan $\left(\frac{\pi}{3}\right)$ $\left(\because \tan\left(\frac{\pi}{3}\right) = -\sqrt{3}\right)$
$\Rightarrow$ tan x = tan $\left(-\frac{\pi}{3}\right)$ $(\because -\tan(\theta) = \tan(-\theta))$
$\Rightarrow$ x = $-\frac{\pi}{3}$

25.

(b) $\frac{\pi}{6}$

Explanation: sin^{-1} $\frac{1}{2}$ = α, say $\Rightarrow$ sin α = $\frac{1}{2}$ = sin $\frac{\pi}{6}$
$\Rightarrow \alpha = \frac{\pi}{6} \in \left|-\frac{\pi}{2}, \frac{\pi}{2}\right|$
$\Rightarrow$ Principal value of sin^{-1} $\frac{1}{2}$ is $\frac{\pi}{6}$.

26.

(b) 0.96

Explanation: Let sin^{-1} (0.8) = $\theta \Rightarrow$ sin θ = 0.8
$\Rightarrow$ cos θ = $\sqrt{1-\sin^2\theta} \Rightarrow$ cos θ = $\sqrt{1-(0.8)^2}$
$\Rightarrow$ cos θ = $\sqrt{1-0.64} \Rightarrow$ cos θ = $\sqrt{0.36}$
$\Rightarrow$ cos θ = 0.6
$\therefore$ sin(2 sin^{-1} (0.8)) = sin 2θ = 2 sin θ cos θ = 2 $\times$ 0.8 $\times$ 0.6 = 0.96

27. **(a)** $\frac{-3\pi}{4}$

Explanation: $\tan^{-1}(-2) + \tan^{-1}(-3)$
as we know; $\tan^{-1}x + \tan^{-1}y = -\pi + tan^{-1}(\frac{x+y}{1-xy})\ if\ xy > 1, x < 0, y < 0$
$= -\pi + \tan^{-1}\left(\frac{-2+(-3)}{1-(-2)(-3)}\right)$
$= -\pi + \tan^{-1}\left(\frac{(-5)}{(-5)}\right) = -\pi + \tan^{-1}(1) = -\pi + \frac{\pi}{4} = \frac{-3\pi}{4}$

28.

(d) [0, π]

Explanation: We know that the principal value branch of cos^{-1}x is [0, π]

29. **(a)** 0.96

Explanation: Let sin^{-1} (0.6) = θ, i.e, sin θ =0.6
Now, sin (2θ) = 2 sinθ cosθ = 2 (0.6) (0.8) = 0.96

30. **(a)** x $\in$ (2 - $\sqrt{9-2\pi}$, 2 + $\sqrt{9-2\pi}$)

Explanation: $\because$ 1 rad = 57.75^o $\Rightarrow$ 5 rad = 288.75^o
$\Rightarrow$ $\frac{3\pi}{2}$ < 5 < $\frac{5\pi}{2}$ $\Rightarrow$ sin^{-1} (sin 5) = 5 - 2π
Now, $\because$ sin^{-1} (sin 5) > x^2 - 4x
$\Rightarrow$ 5 - 2π > x^2 - 4x $\Rightarrow$ x^2 - 4x + (2π - 5) < 0
$\Rightarrow$ 2 - $\sqrt{9-2\pi}$ < x < 2 + $\sqrt{9-2\pi}$) ($\because$ x^2 - 4x + (2π - 5) = 0 $\Rightarrow$ x = 2 $\pm\sqrt{9-2\pi}$)
$\Rightarrow$ x $\in$ (2 - $\sqrt{9-2\pi}$, 2 + $\sqrt{9-2\pi}$)

31. **(a)** $[2\pi, 3\pi]$

Explanation: $[2\pi, 3\pi]$

32.

(b) $\frac{3\pi}{4}$

Explanation: Let the principle value be given by x
also, let x = cos^{-1} $\left(\frac{-1}{\sqrt{2}}\right)$
$\Rightarrow \cos x = \frac{-1}{\sqrt{2}}$
$\Rightarrow \cos x = -\cos\left(\frac{\pi}{4}\right)$ $\left(\because \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}\right)$
$\Rightarrow$ cos x = cos $(\pi - \frac{\pi}{4})$ {$\because$ -cos (θ) = cos $(\pi - \theta)$}
$\Rightarrow x = \frac{3\pi}{4}$

33.

(b) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ - {0}

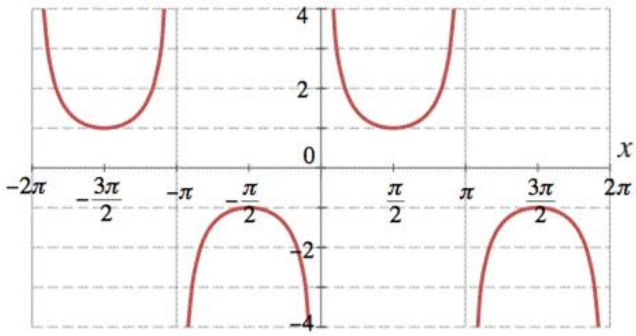
Explanation: To Find: The range of $\text{coses}^{-1}(x)$

Here, the inverse function is given by $y = f^{-1}(x)$

The graph of the function $\text{coses}^{-1}(x)$ can be obtained from the graph of

$Y = \text{coses}^{-1}(x)$ by interchanging x and y axes.i.e, if a, b is a point on Y = cosec x then b, a is the point on the function $y = \text{coses}^{-1}(x)$

Below is the Graph of the range of $\text{coses}^{-1}(x)$



From the graph, it is clear that the range of $\text{coses}^{-1}(x)$ is restricted to interval $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ - {0}

34.

(b) $\frac{-\pi}{2}$

Explanation: Let $\text{cosec}^{-1}(-1) = \alpha \Rightarrow \text{cosec}\,\alpha = -1 = \text{cosec}\left(\frac{-\pi}{2}\right)$

$\Rightarrow \alpha = -\frac{\pi}{2} \in \left|-\frac{\pi}{2}, \frac{\pi}{2}\right| - \{0\}$

$\therefore$ Principal value of $\text{cosec}^{-1}(-1)$ is $\frac{-\pi}{2}$.

35.

(c) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

Explanation: To Find: The range of $\sin^{-1}x$

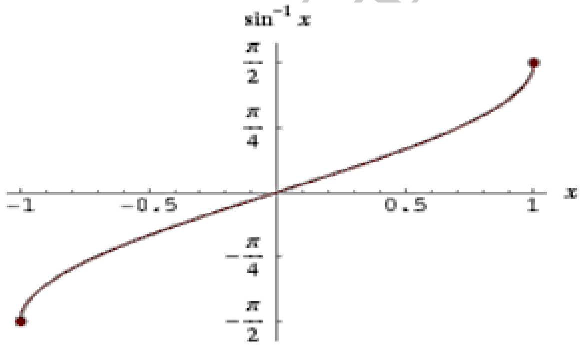
Here, the inverse function is given by $y = f^{-1}(x)$

The graph of the function $y = \sin^{-1}(x)$ can be obtained from the graph of

Y = sin x by interchanging x and y axes.i.e, if (a, b) is a point on Y = sin x then (b, a) is

The point on the function $y = \sin^{-1}(x)$

Below is the Graph of range of $\sin^{-1}(x)$



From the graph, it is clear that the range $\sin^{-1}(x)$ is restricted to the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

36.

(d) 1

Explanation: $\sin\left[\frac{\pi}{3} + \sin^{-1}\left(\frac{1}{2}\right)\right]$

$= \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right) = \sin\left(\frac{\pi}{2}\right) = 1$

37.

(c) $\frac{\pi}{12}$

Explanation: $\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

$=\cos^{-1}\left(\cos\frac{\pi}{3}\right)-\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\ \left[\sin^{-1}(-\theta)=-\sin^{-1}\theta\right]$

$=\frac{\pi}{3}-\sin^{-1}\left(\sin\frac{\pi}{4}\right)=\frac{\pi}{3}-\frac{\pi}{4}=\frac{\pi}{12}$

38. **(a)** [-1, 2)

Explanation: Domain of $\sin^{-1} x$ is [-1, 1].

$\therefore$ Domain of $\sin^{-1}[x]$ is $\{x : -1 \leq [x] \leq 1\}$

But $[x]=\begin{cases}-1 & \forall \quad -1\leq x<0\\ 0 & \forall \quad 0\leq x<1\\ 1 & \forall \quad 1\leq x<2\end{cases}$

$\therefore$ Domain of $\sin^{-1}[x]$ is [-1, 2)

39.

(b) $-\frac{24}{25}$

Explanation: put $\cos^{-1}\left(-\frac{3}{5}\right)=\theta \implies \cos\theta=-\frac{3}{5}$ therefore the given expression is

$\sin 2\theta=2sin\theta cos\theta=\frac{2.4}{5}.\left(-\frac{3}{5}\right)=\left(-\frac{24}{25}\right).$

40. **(a)** $-\frac{\pi}{10}$

Explanation: $\sin^{-1}\left(\cos\frac{40\pi+3\pi}{5}\right)$

$=\sin^{-1}\cos\left(8\pi+\frac{3\pi}{5}\right)$

$=\sin^{-1}\left(\cos\frac{3\pi}{5}\right)$

$=\sin^{-1}\left(\sin\left(\frac{\pi}{2}-\frac{3\pi}{5}\right)\right)$

$=\sin^{-1}\left(\sin\left(-\frac{\pi}{10}\right)\right)=-\frac{\pi}{10}$

41. **(a)** [–1, 1]

Explanation: The domain of cos is R and the domain of $\sin^{-1}$ is [–1, 1]. Therefore, the domain of $\cos x + \sin^{-1}x$ is $R \cap [-1,1]$, i.e., [–1, 1].

42.

(d) $\frac{\pi}{3}$

Explanation: $\sin^{-1}\sin\left(-600\times\frac{\pi}{180}\right)=\sin^{-1}\sin\left(\frac{-10\pi}{3}\right)$

$=\sin^{-1}\left[-\sin\left(4\pi-\frac{2\pi}{3}\right)\right]=\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

$=\sin^{-1}\left(\sin\left(\pi-\frac{\pi}{3}\right)\right)=\sin^{-1}\left(\sin\frac{\pi}{3}\right)=\frac{\pi}{3}$

43. **(a)** $-\frac{\pi}{3}$

Explanation: $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)=\sin^{-1}\left(-\sin\frac{\pi}{3}\right)=-\sin^{-1}\left(\sin\frac{\pi}{3}\right)=-\frac{\pi}{3}$

44.

(d) $\frac{19\pi}{12}$

Explanation: $\sin^{-1}\left(-\frac{1}{2}\right)+\cos^{-1}\left(-\frac{1}{2}\right)+\cot^{-1}(-\sqrt{3})+\operatorname{cosec}^{-1}(\sqrt{2})+\tan^{-1}(-1)+\sec^{-1}(\sqrt{2})$

$=-\sin^{-1}\left(-\frac{1}{2}\right)+\pi-\cos^{-1}\left(\frac{1}{2}\right)+\pi-\cot^{-1}(\sqrt{3})+\operatorname{cosec}^{-1}(\sqrt{2})-\tan^{-1}(1)+\sec^{-1}(\sqrt{2})$

$=-\frac{\pi}{6}+\pi-\frac{\pi}{3}+\pi-\frac{\pi}{6}+\frac{\pi}{4}-\frac{\pi}{4}+\frac{\pi}{4}$

$=-\frac{2\pi}{3}+\frac{9\pi}{4}=\frac{19\pi}{12}$

45. **(a)** $2\cos^{-1}x$

Explanation: $\cos^{-1}(2x^2-1)$

Put $x=\cos\alpha \Rightarrow \alpha=\cos^{-1}x$

$\therefore \cos^{-1}(2x^2-1)=\cos^{-1}(2\cos^2\alpha-1)=\cos^{-1}(\cos 2\alpha)$

$[\because 0 \leq x \leq 1 \Rightarrow \cos\left(-\frac{\pi}{2}\right) \leq \cos\alpha \leq \cos 0 \Rightarrow -\frac{\pi}{2} \leq \alpha \leq 0 \Rightarrow -\pi \leq 2\alpha \leq 0]$

= 2α = 2 $\cos^{-1}$ x

46.

(d) $\frac{\pi}{6}$

Explanation: Let $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ = $\theta \Rightarrow$ sec θ = $\frac{2}{\sqrt{3}}$

We know that, the range of principal value branch of $\sec^{-1}\theta$ is $[0, \pi]$ - $\left\{\frac{\pi}{2}\right\}$

$\therefore$ sec θ = $\frac{2}{\sqrt{3}}$ = sec $\frac{\pi}{6}$

$\Rightarrow \theta$ = $\frac{\pi}{6}$, where $\theta \in [0, \pi]$ - $\left\{\frac{\pi}{2}\right\}$.

$\Rightarrow \sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ = $\frac{\pi}{6}$

47.

(b) x

Explanation: Let $\cos^{-1}$ x = θ

$\Rightarrow$ x = cos $\theta \Rightarrow$ sec θ = $\frac{1}{x} \Rightarrow$ tan θ = $\sqrt{\sec^2\theta - 1}$

$\Rightarrow$ tan θ = $\sqrt{\frac{1}{x^2} - 1} \Rightarrow$ tan θ = $\frac{1}{x}\sqrt{1 - x^2}$

Now, sin [$\cot^{-1}$ {tan ($\cos^{-1}$x))] = sin [$\cot^{-1}$ {tan θ}]

= sin[$\cot^{-1}\left\{\frac{1}{x}\sqrt{1 - x^2}\right\}$]

Again, let x = sin α = sin[$\cot^{-1}\left\{\frac{1}{\sin\alpha}\sqrt{1 - \sin\alpha^2}\right\}$]

= sin[$\cot^{-1}\left\{\frac{\cos\alpha}{\sin\alpha}\right\}$] = sin[$\cot^{-1}$(cot α)]

= sin α = x

48.

(d) $\frac{\pi}{6}$

Explanation: Let the principle value be given by x

Now, let x = cosec^{-1}(2)

$\Rightarrow$ cosec x = 2

$\Rightarrow$ cosec x = cosec $\left(\frac{\pi}{6}\right)$ $\left(\because \cos\left(\frac{\pi}{6}\right) = 2\right)$

$\Rightarrow x = \frac{\pi}{6}$

49. **(a)** [1, 2]

Explanation: f(x) = [1, 2]

$\Rightarrow 0 \leq$ x - 1 $\leq$ 1 [Since, $\sqrt{x-1} \geq 0$ and -1 $\leq \sqrt{x-1} \leq 1$]

$\Rightarrow 1 \leq x \leq 2$

$\therefore$ x $\in$ [1, 2]

50. **(a)** $\frac{23\pi}{12}$

Explanation: $\because$ $\tan^{-1}(\sqrt{3})$ + $\cot^{-1}$(-1) + $\sec^{-1}\left(-\frac{2}{\sqrt{3}}\right)$

$= \frac{\pi}{3} + \pi$ - $\cot^{-1}$ (1) + π - $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

$= \frac{\pi}{3} + 2\pi - \frac{\pi}{4} - \frac{\pi}{6} = \frac{7\pi}{3} - \frac{5\pi}{12}$

$= \frac{28\pi - 5\pi}{12} = \frac{23\pi}{12}$

51.

(b) R - (-1, 1)

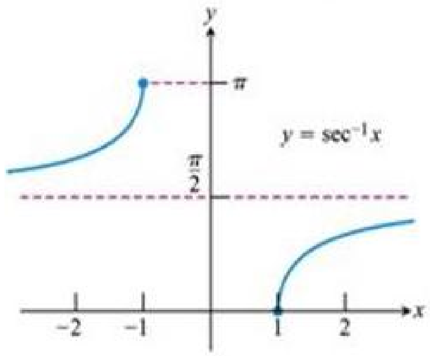
Explanation: We have to find: The range of $\sec^{-1}$(x)

Here, the inverse function is given by y = f^{-1}(x)

The graph of the function y = $\sec^{-1}$(x) can be obtained from the graph of

Y = sec x by interchanging x and y axes.i.e, if a, b is a point on Y = sec x then b, a is the point on the function y = $\sec^{-1}$(x)

Below is the Graph of the range of $\sec^{-1}$(x)



From the graph, it is clear that the domain of $\sec^{-1}(x)$ is a set of all real numbers excluding -1 and 1 i.e, R - (-1, 1).
Which is the required solution.

52. **(a)** 4

Explanation: We have

$\sin^{-1}(x^2 - 7x + 12) = n\pi \Rightarrow x^2 - 7x + 12 = \sin(n\pi), n \in Z$

$\Rightarrow x^2 - 7x + 12 = 0 \Rightarrow x^2 - 3x - 4x + 12 = 0$

$\Rightarrow x(x - 3) - 4(x - 3) = 0 \Rightarrow (x - 3)(x - 4) = 0$

$\Rightarrow x = 3, 4$

53.

(b) $\frac{-\pi}{10}$

Explanation: We have,

$\sin^{-1}\left[\cos\left(\frac{33\pi}{5}\right)\right]$

$= \sin^{-1}\left[\cos\left(6\pi + \frac{3\pi}{5}\right)\right]$

$= \sin^{-1}\left[\cos\left(\frac{3\pi}{5}\right)\right]$ [Since, $\cos(2n\pi + \theta) = \cos\theta$]

$= \sin^{-1}\left[\cos\left(\frac{\pi}{2} + \frac{\pi}{10}\right)\right]$

$= \sin^{-1}\left[\sin\left(-\frac{\pi}{10}\right)\right]$ [Since, $\sin^{-1}(-x) = -\sin^{-1}x$]

$= -\frac{\pi}{10}$ [Since, $\sin^{-1}(\sin x) = x, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$]

54.

(b) [-1, 2)

Explanation: Let $f(x) = \cos^{-1}[x]$

Now, domain of $g(x) = \cos^{-1}x$ is the set

$-1 \leq x \leq 1\} = [-1, 1]$

$\therefore$ Domain of given function is $\{x : -1 \leq [x] \leq 1\}$

$$[x] = \begin{cases} -1 & \text{if} \quad -1 \leq x \leq 0 \\ 0 & \text{if} \quad 0 \leq x \leq 1 \\ 1 & \text{if} \quad 1 \leq x < 2 \end{cases}$$

$\therefore$ Domain of $\cos^{-1}[x]$ is [-1, 2).

55. **(a)** $\frac{7\pi}{18}$

Explanation: $\sin^{-1}(\cos\frac{\pi}{9}) = \sin^{-1}(\sin\left(\frac{\pi}{2} - \frac{\pi}{9}\right)) = \sin^{-1}(\sin\frac{7\pi}{18}) = \frac{7\pi}{18}$

56.

(b) All of these

Explanation: $3\sin x + 4\cos x = y^2 - 2y + 6 = (y - 1)^2 + 5$

Which is possible only when $y - 1 = 0 \Rightarrow y = 1$ ($\because$ Maximum value of LHS = 5 and Minimum value of RHS = 5)

$\therefore 3\sin x + 4\cos x = 5$

Put $3 = r\cos\theta$, $4 = r\sin\theta$ such that $9 + 16 = r^2$

$\Rightarrow r^2 = 25 \Rightarrow r = 5$

Also tan $\theta = \frac{4}{3} \Rightarrow \theta = \tan^{-1}\left(\frac{4}{3}\right)$

$\Rightarrow$ r cos θ sin x + r sin θ cos x = 5

$\Rightarrow$ r sin (x + θ) = r $\Rightarrow$ sin (x + θ) = 1

$\Rightarrow x + \theta = \sin^{-1}(1) = \frac{\pi}{2}$

$\Rightarrow x = \frac{\pi}{2} - \theta = \frac{\pi}{2} - \tan^{-1}\left(\frac{4}{3}\right)$

$\therefore xy = \frac{\pi}{2} - \tan^{-1}\left(\frac{4}{3}\right)$

So all option are true.

57. **(a)** $\frac{13\pi}{15}$

Explanation: $\cos^{-1}\left[\cos\left(\left(-\frac{17}{15}\right)\pi\right)\right] = \cos^{-1}\left[\cos\left(\frac{17\pi}{15}\right)\right]$

$= \cos^{-1}\left[\cos\left(2\pi - \frac{13\pi}{15}\right)\right] = \cos^{-1}\left[\cos \frac{13\pi}{15}\right] = \frac{13\pi}{15}$

58.

(d) [1, 2]

Explanation: Let, $f(x) = \cos^{-1}(2x - 3)$

$-1 \leq 2x - 3 \leq 1$

$\Rightarrow 2 \leq 2x \leq 4$

$\Rightarrow 1 \leq x \leq 2$

$\therefore x \in [1, 2]$ or domain of x is [1, 2].

59.

(d) $-\frac{\pi}{2}$

Explanation: $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$

$= \tan^{-1}\left(\tan\frac{\pi}{3}\right) - \pi + \cot^{-1}\left(\cot\frac{\pi}{6}\right)$

$= \frac{\pi}{3} - \left(\pi - \frac{\pi}{6}\right) = -\frac{\pi}{2}$ $[\cot^{-1}(-\theta) = \pi - \cot^{-1}\theta]$

60. **(a)** $\frac{5\pi^2}{4}$ and $\frac{\pi^2}{8}$

Explanation: We have

$(\sin^{-1}x)^2 + (\cos^{-1}x)^2 = (\sin^{-1}x + \cos^{-1}x)^2 - 2\sin^{-1}x\cos^{-1}x$

$= \frac{\pi^2}{4} - 2\sin^{-1}x\left(\frac{\pi}{2} - \sin^{-1}x\right)$

$= \frac{\pi^2}{4} - \pi\sin^{-1}x + 2(\sin^{-1}x)^2$

$= 2\left[\left(\sin^{-1}x\right)^2 - \frac{\pi}{2}\sin^{-1}x + \frac{\pi^2}{8}\right]$

$= 2\left[\left(\sin^{-1}x - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{16}\right]$

Thus, the least value is $2\left(\frac{\pi^2}{16}\right)$ i.e. $\frac{\pi^2}{8}$ and the Greatest value is $2\left[\left(\frac{-\pi}{2} - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{16}\right]$ i.e. $\frac{5\pi^2}{4}$.

61. **(a)** $\sqrt{\frac{2}{3}}$

Explanation: $\sin\left[\cot^{-1}\left(\cos\frac{\pi}{4}\right)\right] = \sin\left[\cot^{-1}\frac{1}{\sqrt{2}}\right] = \sin\left[\sin^{-1}\sqrt{\frac{2}{3}}\right] = \sqrt{\frac{2}{3}}$.

Which is the required solution.

62.

(b) $\frac{3\pi}{4}$

Explanation: Let the principle value be given by x

also, let $x = \cot^{-1}(-1)$

$\Rightarrow \cot x = -1$

$\Rightarrow \cot x = -\cot\left(\frac{\pi}{4}\right)\left(\because \cot\left(\frac{\pi}{4}\right) = 1\right)$

$\Rightarrow \cot x = \cot\left(\pi - \frac{\pi}{4}\right)(\because -\cot(\theta) = \cot(\pi - \theta))$

$\Rightarrow x = \frac{3\pi}{4}$

63.

(c) { -1 ,1}

Explanation: Since, Domain of $\sin^{-1}x$ is [-1,1] and domain of $\sec^{-1}x$ is R - (-1,1), D_f = { -1,1 }.

64. **(a)** $\frac{1}{2}$ sin-1 $\frac{3}{4}$

Explanation: We have

sin (π cos x) = cos (π sin x)

$\Rightarrow$ sin (π cos x) = sin ($\frac{\pi}{2} \pm \pi$ sin x)

$\Rightarrow \pi$ cos x) = $\frac{\pi}{2} \pm \pi$ sin x $\Rightarrow \pi$ cos x $\mp \pi$ sin x = $\frac{\pi}{2}$

$\Rightarrow$ cos x $\mp$ sin x = $\frac{1}{2}$

Squaring both sides, we get

1 $\pm$ sin 2x = $\frac{1}{4} \Rightarrow \pm$ sin 2x = $\frac{1}{4}$ - 1

$\Rightarrow \pm$ sin 2x = $-\frac{3}{4} \Rightarrow$ sin 2x = $\pm \frac{3}{4}$

$\Rightarrow$ x = $\frac{1}{2}$ sin-1 $\left(\pm\frac{3}{4}\right) \Rightarrow$ x = $\pm \frac{1}{2}$ sin-1 $\left(\frac{3}{4}\right)$

65.

(d) $\frac{2\pi}{9}$

Explanation: $\cos^{-1}$ (cos (680°))

= $\cos^{-1}$ [cos (720° – 40°)]

= $\cos^{-1}$ [cos (– 40°)]

= $\cos^{-1}$ [cos (40°)]

= 40°

= $\frac{2\pi}{9}$.

66.

(b) $\frac{5\pi}{6}$

Explanation: Let the principle value be given by x

also, let x = cot-1 $(-\sqrt{3})$

$\Rightarrow$ cot x = $-\sqrt{3}$

$\Rightarrow$ cot x = -cot $\left(\frac{\pi}{6}\right)$ ($\because$ cot$\left(\frac{\pi}{6}\right) = \sqrt{3}$)

$\Rightarrow$ cot x = cot $\left(\pi - \frac{\pi}{6}\right)$ ($\because$ - cot (θ) = cot ($\pi - \theta$))

$\Rightarrow x = \frac{5\pi}{6}$

67.

(b) None of these

Explanation: We know that $cos : [0,1] \rightarrow [-1,1]$ is bijective function

$\Rightarrow cos^{-1} : [-1,1] \rightarrow [0,1]$ is inverse of cos function.

.$\Rightarrow \cos(\cos^{-1}x) = x$ when $x \in [-1,1]$

here, $cos(\cos^{-1}\frac{7}{25}) = \frac{7}{25}$, $\frac{7}{25} \in [-1,1]$.

68.

(c) 0

Explanation: $\cot^{-1}21 + \cot^{-1}13 - \cot^{-1}8$

$\Rightarrow \tan^{-1}\left(\frac{1}{21}\right) + \tan^{-1}\left(\frac{1}{13}\right) - \tan^{-1}\left(\frac{1}{8}\right)$

$\Rightarrow \tan^{-1}\left[\frac{\frac{1}{21}+\frac{1}{13}}{1-\frac{1}{21}\cdot\frac{1}{13}}\right] - \tan^{-1}\left(\frac{1}{8}\right) \qquad \left(\frac{1}{21}\cdot\frac{1}{13} < 1\right)$

$\Rightarrow \tan^{-1}\left[\frac{\frac{34}{21.13}}{\frac{272}{21.13}}\right] - \tan^{-1}\left(\frac{1}{8}\right)$

$\Rightarrow \tan^{-1}\left(\frac{34}{272}\right) - \tan^{-1}\left(\frac{1}{8}\right)$

$\Rightarrow \tan^{-1}\left(\frac{1}{8}\right) - \tan^{-1}\left(\frac{1}{8}\right) = 0$.

69.

(c) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Explanation: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

70.

(b) $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

Explanation: $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

71.

(d) $\frac{2\pi}{3}$

Explanation: $\sec^{-1}(\sec \frac{4\pi}{3}) = \sec^{-1}\left(\sec\left(\pi + \frac{\pi}{3}\right)\right)$

$= \sec^{-1}(-\sec \frac{\pi}{3}) = \sec^{-1}(-2) = \pi - \sec^{-1} 2$

$= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

72.

(b) $[-\tan 1, \infty)$

Explanation: We have

$[\tan^{-1}x]^2 - 2[\tan^{-1}x] - 3 \leq 0$

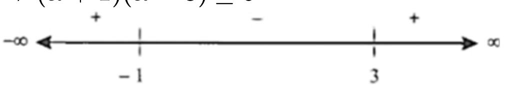
Put $[\tan^{-1}x] = \alpha$

$\Rightarrow \alpha^2 - 2\alpha - 3 \leq 0$

$\Rightarrow \alpha^2 + \alpha - 3\alpha - 3 \leq 0$

$\Rightarrow \alpha(\alpha + 1) - 3(\alpha + 1) \leq 0$

$\Rightarrow (\alpha + 1)(\alpha - 3) \leq 0$



$\Rightarrow -1 \leq \alpha \leq 3$

$\Rightarrow -1 \leq [\tan^{-1}x] \leq 3$

$\Rightarrow -1 \leq \tan^{-1}x < 4$

$\Rightarrow \tan(-1) \leq x < \tan 4 < \infty$

$\Rightarrow -\tan 1 \leq x < \infty$

$\Rightarrow x \in [-\tan 1, \infty)$

73.

(b) $\sqrt{3}$

Explanation: $\cot\left[\frac{1}{2} \sin^{-1} \frac{\sqrt{3}}{2}\right]$

$= \cot\left[\frac{1}{2} \times \frac{\pi}{3}\right] = \cot^{-1}\left(\frac{\pi}{6}\right) = \sqrt{3}$

74. **(a)** $\frac{\sqrt{1-x^2}}{x}$

Explanation: Let $\sin^{-1} x = \theta$, then $\sin\theta = x$

$\Rightarrow \operatorname{cosec} \theta = \frac{1}{x} \Rightarrow \operatorname{cosec}^2\theta = \frac{1}{x^2}$

$\Rightarrow 1 + \cot^2 \theta = \frac{1}{x^2}$

$\Rightarrow \cot \theta = \frac{\sqrt{1-x^2}}{x}$

75. **(a)** 2

Explanation: Since, $-1 \leq \sin^{-1} x \leq 1$ and given $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$

$\Rightarrow \sin^{-1}x = \sin^{-1}y = \sin^{-1}z = \frac{\pi}{2}$

$\Rightarrow x = y = z = 1$

Also, $\because f(1) = 1$

and $f(p + q) = f(p) \cdot f(q)\ \forall\ p, q \in R$, then

$\Rightarrow f(2) = f(1 + 1) = f(1) \cdot f(1) = 1 \times 1 = 1$

$\Rightarrow f(3) = f(2 + 1) = f(2) \cdot f(1) = 1 \times 1 = 1$

Now, $x^{f(1)} + y^{f(2)} + z^{f(3)} = \frac{x+y+z}{x^{f(1)}+y^{f(2)}+z^{f(3)}} = 1 + 1 + 1 - \frac{1+1+1}{x^1+y^1+z^1}$

$= 3 - \frac{3}{1+1+1} = 3 - \frac{3}{3} = 3 - 1 = 2$