Solution

CET25M12 LINEAR PROGRAMMING

Class 12 - Mathematics

1.

(c) infinite

Explanation: In a LPP, if the objective f^n Z = ax + by has the maximum value on two corner point of the feasible region then every point on the line segment joining these two points gives the same maximum value.
hence, Z_{max} occurs at infinite no of times.

2.

(c) a = 2b
Explanation: The maximum value of 'z' occurs at (2, 4) and (4, 0)
$\therefore$ Value of z at (2, 4) = value of z at (4, 0)
a(2) + b(4) = a(4) + b(0)
2a + 4b = 4a + 0
4b = 4a - 2a
4b = 2a
a = 2b

3. **(a)** q = 3p
Explanation: Since Z occurs maximum at (15, 15) and (0, 20), therefore, 15p + 15q = 0p + 20q $\Rightarrow$ q = 3p.

4.

(c) not in the region
Explanation: Since (0, 0) does not satisfy x + y $\geq$ 1
i.e., 0 + 0 $\neq$ 1
$\Rightarrow$ (0, 0) not lie in feasible region represented by x + y $\geq$ 1.

5.

(d) Option (c)
Explanation: If a LPP admits two optimal solutions it has an infinite solution.

6.

(c) {X: |X| = 5}
Explanation: |x| = 5 is not a convex set as any two points from negative and positive x -axis if joined will not lie in set.

7.

(b) open half plane not containing the origin
Explanation: open half plane not containing the origin
On putting x = 0, y = 0 in the given inequality, we get 0 > 5, which is absurd.
Therefore, the solution set of the given inequality does not include the origin.
Thus, the solution set of the given inequality consists of the open half plane not containing the origin.

8. **(a)** given by corner points of the feasible region
Explanation: It is known that the optimal value of the objective function is attained at any of the corner point. Thus, the optimal value of the objective function is attained at the points given by corner points of the feasible region.

9. **(a)** Z has no maximum value
Explanation: Objective function is Z = - x + 2 y ……………………(1).
The given constraints are : x ≥ 3, x + y ≥ 5, x + 2y ≥ 6, y ≥ 0.

Corner points	Z = - x + 2y
D(6,0)	-6
A(4,1)	-2
B(3,2)	1

Here, the open half plane has points in common with the feasible region.
Therefore, Z has no maximum value.

10.

(d) $p = \frac{q}{2}$
Explanation: We have Z = px + qy , At (3, 0) Z = 3p ……………………………….(1)
At (1 , 1) Z = p + q …………………………(2) Therefore , from (1) and (2) : We have : p = q/2 .

11.

(c) 1700
Explanation: Here , Maximize Z = 50x+60y , subject to constraints x +2 y ≤ 50 , x + y ≥ 30, x, y ≥ 0.

Corner points	Z = 50x +60 y
P(50 ,0)	2500
Q(0 , 30)	1800
R(10, 20)	1700

Hence the minimum value is 1700

12.

(b) q = 3p
Explanation: The maximum value of Z is unique.
It is given that the maximum value of Z occurs at two points (3,4) and (0,5)
∴ Value of Z at (3, 4) = Value of Z at (0, 5)
⇒ p(3) + q(4) = p(0) + q(5)
⇒ 3p + 4q = 5q
⇒ q = 3p

13. **(a)** Minimum value of Z is -5
Explanation:

Corner points	**Value of Z = 2x - y + 5**
A(0, 10)	Z = 2(0) - 10 + 5 = -5 (Minimum)
B(12, 6)	Z = 2(12) - 6 + 5 = 23
C(20, 0)	Z = 2(20) - 0 + 5 = 45 (Maximum)
O(0, 0)	Z = 0(0) - 0 + 5 = 5

So the minimum value of Z is -5.

14.

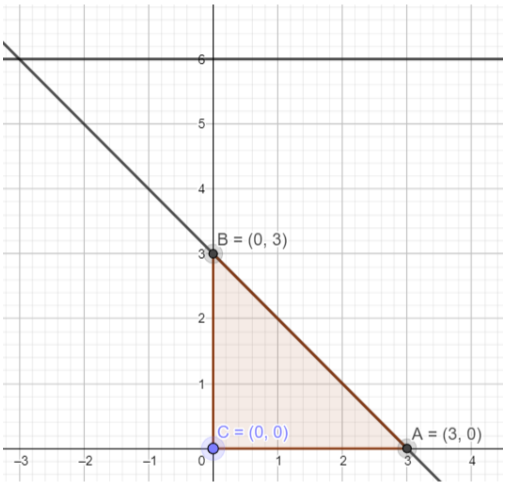
(c) $X = \lambda X_1 + (1-\lambda)X_2, 0 \leq \lambda \leq 1$ gives an optimal solution
Explanation: X = $\lambda X_1 + (1-\lambda)X_2$, $0 \leq \lambda \leq 1$ gives an optimal solution
A set 'A' is convex if, for any two points, x_1, $x_1 \in$ A, and $\lambda \in [0,1]$ imply that $\lambda x_1 + (1-\lambda)x_2 \in A$
since, here X_1 and X_2 are optimal solutions
Therefore, their convex combination will also be an optimal solution
Thus, X = $\lambda X_1 + (1-\lambda)X_2$, $0 \leq \lambda \leq 1$ gives an optimal solution.

15.

(b) bounded in first quadrant
Explanation: Converting the given inequations into equations, we obtain
y = 6, x + y = 3, x = 0 and y = 0, y = 6 is the line passing through (0, 6) and parallel to the X axis.The region below the line y = 6 will satisfy the given inequation.
The line x + y = 3 meets the coordinate axis at A(3, 0) and B(0, 3). Join these points to obtain the line x + y = 3 Clearly, (0, 0) satisfies the inequation x + y $\leq$3 . So, the region in x y -plane that contains the origin represents the solution set of the given equation.

The region represented by $x \geq 0$ and $y \geq 0$:

Since every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations.



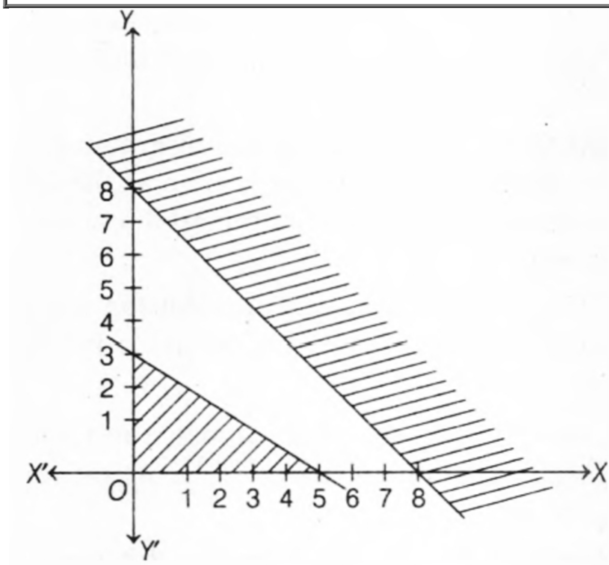
16. **(a)** no feasible solution

Explanation: Table for equation x + y = 8 is

x	0	8
y = 8 - x	8	0

Table for equation 3x + 5y = 15 is

x	0	5
y = $\frac{15-3x}{5}$	3	0



It can be concluded from the graph, that there is no point, which can satisfy all the constraints simultaneously. Therefore, the problem has no feasible solution.

17.

(b) the problem is to be re-evaluated

Explanation: The optimisation of the objective function of a LPP is governed by the constraints. Therefore, if the constraints in a linear programming problem are changed, then the problem needs to be re-evaluated.

18.

(c) $\{(x, y) : x \geq 2, y \leq 4\}$

Explanation: $\{(x, y) : x \geq 2, y \leq 4\}$ is the region between two parallel lines, so any line segment joining any two points in it lies in it. Hence, it is a convex set.

19.

(d) Minimum Z = 300 at (60, 0)

Explanation: Objective function is Z = 5x + 10 y ……………………(1).

The given constraints are : x + 2y ≤ 120, x + y ≥ 60, x – 2y ≥ 0, x, y ≥ 0 .

The corner points are obtained by drawing the lines x+2y =120, x+y = 60 and x-2y = 0. The points so obtained are (60,30), (120,0), (60,0) and (40,20)

Corner points	Z = 5x + 10y
D(60 ,30)	600
A(120,0)	600
B(60,0)	300……………………..(Min.)
C(40,20)	400

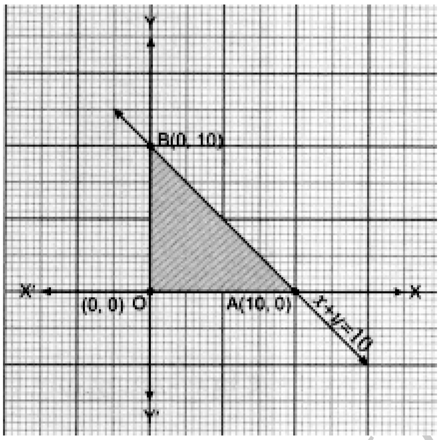
Here , Z = 300 is minimum at (60, 0).

20.

(c) bounded

Explanation: A feasible region of a system of linear inequalities is said to be bounded, if it can be enclosed within a circle.

21. **(a)** 40

Explanation: 

Feasible region is shaded region shown in figure with corner points 0(0, 0), A(10, 0), B(0, 10), Z(0, 0) = 0, Z (10, 0) = 40 ⟶ maximum Z (0, 10) = 30

22.

(b) (0, 3)

Explanation: (0, 3) satisfy the equation 2x + 3y $\leq$ 12

$2 \times 0 + 3 \times 3 \leq 12$

$9 \leq 12$

But (3, 3), (4, 3), (0, 5) does not satisfy 2x + 3y $\leq$ 12.

23.

(d) 47

Explanation: We have , Maximise the function Z = 11x + 7y, subject to the constraints: x ≤ 3, y ≤ 2,x ≥ 0, y ≥ 0.

Corner points	Z = 11x +7 y
C(0, 0)	0
B (3,0)	33
D(0,2)	14
A(3, 2)	47

Hence the function has maximum value of 47

24.

(c) 42

Explanation: Here , maximize Z = 11x + 7y , subject to the constraints :2x + y ≤ 6, x ≤ 2, x ≥ 0, y ≥ 0.

Corner points	Z = 11x +7 y
C(0, 0)	0
B (2,0)	22
D(2,2)	36
A(0,6)	42

Hence the maximum value is 42

25. **(a)** 4

Explanation: According to the question, maximize , Z = 3x + 4y, subject to the constraints: $x + y \leq 1, x \geq 0, y \geq 0$.

Corner points	Z = 3x +4 y
C(0, 0)	0
B (1,0)	3
D(0,1)	4

Hence the maximum value is 4

26. **(a)** 1260

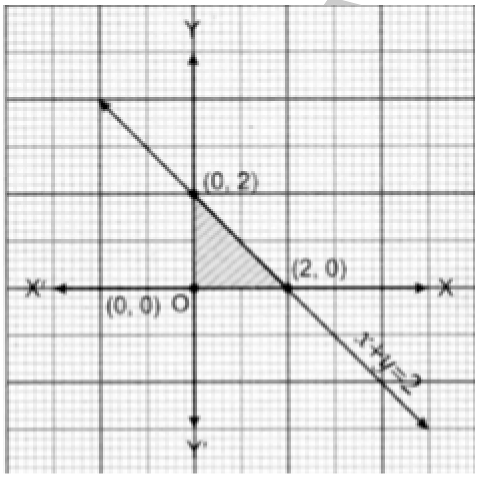
Explanation: We have , Maximize Z = 100x + 120y , subject to constraints $2x + 3y \leq 30, 3x + y \leq 17, x \geq 0, y \geq 0$.

Corner points	Z = 100x +120 y
P(0 , 0)	0
Q(3 , 8)	1260………………….(Max.)
R(0, 10)	1200
S(17/3 , 0)	1700/3

Hence the maximum value is 1260

27.

(c) 4

Explanation: 

Feasible region is shaded region with corner points (0, 0), (2, 0) and (0, 2).

Z(0, 0) = 0

Z (2, 0) = 4 ⟵ maximum

Z(0, 2) = - 2

Z_{max} = 4 and obtained at (2, 0)

28.

(d) (40,15)

Explanation: We need to maximize the function z = x + y Converting the given inequations into equations, we obtain

x + 2y = 70, 2x + y = 95, x = 0 and y = 0

Region represented by $x + 2y \leq 70$:

The line x + 2y = 70 meets the coordinate axes at A(70, 0) and B(0, 35) respectively. By joining these points we obtain the line x + 2y = 70. Clearly (0, 0) satisfies the inequation x + 2y $\leq$ 70 . So, the region containing the origin represents the solution set of the inequation x + 2y $\leq$ 70.

Region represented by 2x + y $\leq$ 95 :

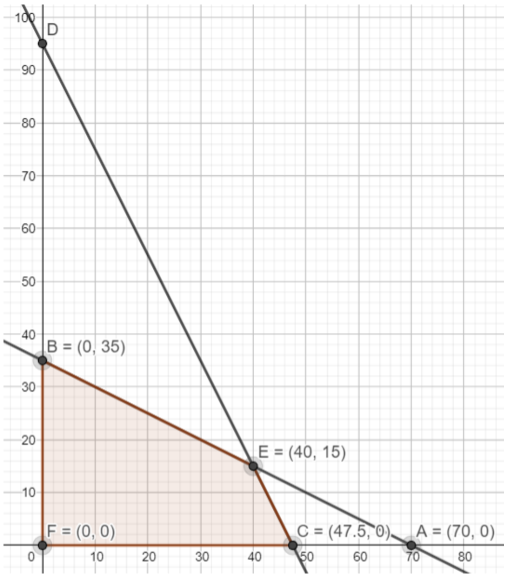
The line 2x + y = 95 meets the coordinate axes at $C\left(\frac{95}{2}, 0\right)$ respectively. By joining these points we obtain the line 2x + y = 95

Clearly (0, 0) satisfies the inequation 2x + y $\leq$ 95 . So, the region containing the origin represents the solution set of the inequation 2x + y $\leq$ 95

Region represented by x $\geq$ 0 and y $\geq$ 0 :

since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations x $\geq$ 0, and y $\geq$ 0

The feasible region determined by the system of constraints x + 2y $\leq$ 70, 2x + y $\leq$ 95, x $\geq$ 0, and y $\geq$ 0 are as follows.



The corner points of the feasible region are O(0, 0), C($\frac{95}{2}$, 0) E(40, 15) and B(0, 35).

The value fo Z at these corner points are as follows.

Corner point : z = x + y

O(0, 0) : 0 + 0 = 0

$C\left(\frac{95}{2}, 0\right) : \frac{95}{2} + 0 = \frac{95}{2}$

E(40, 15) : 40 + 15 = 55

B(0, 35) : 0 + 35 = 35

We see that maximum value of the objective function Z is 55 which is at (40, 15).

29.

(c) 2500

Explanation: Here , Maximize Z = 50x+60y , subject to constraints x +2 y ≤ 50 , x + y ≥ 30, x, y ≥ 0.

Corner points	Z = 50x +60 y
P(50 ,0)	2500
Q(0 , 30)	1800
R(10, 20)	1700

Hence, the maximum value is 2500

30. **(a)** a function to be optimized

Explanation: a function to be optimized

The objective function of a linear programming problem is either to be maximized or minimized i.e. objective function is to be optimized.

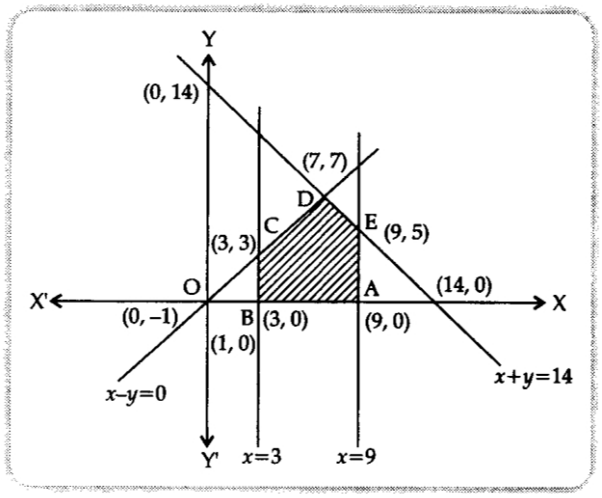
31.

(d) (2, 3)

Explanation: Since (2, 3) does not satisfy $2x + 3y - 12 \leq 0$ as $2 \times 2 + 3 \times 3 - 12 = 4 + 9 - 12 = 1 \neq 0$

32.

(b) 5 corner points including (7, 7) and (3, 3)

Explanation: 

On plotting the constraints x = 3, x = 9, x = y and x + y = 14, we get the following graph. From the graph given below it, clear that feasible region is ABCDEA, including corner points A(9, 0), B(3, 0), C(3, 3), D(7, 7) and E(9, 5).
Thus feasible region has 5 corner points including (7, 7) and (3, 3).

33.

(b) 60

Explanation: Here the objective function is given by : F = 4x +6y .

Corner points	Z = 4x +6 y
(0, 2)	12(Min.)
(3,0)	12.(Min.)
(6,0)	24
(6 , 8)	72
(0 , 5)	30

Maximum of F – Minimum of F = 72 – 12 = 30 .

34.

(b) at any vertex of feasible region

Explanation: In linear programming problem we substitute the coordinates of vertices of feasible region in the objective function and then we obtain the maximum or minimum value. Therefore, the value of objective function is maximum under linear constraints at any vertex of feasible region.

35.

(c) $x_1 = 2$, $x_2 = 6$, Z = 36

Explanation: We need to maximize the function $z = 3x_1 + 5x_2$

First, we will convert the given inequations into equations, we obtain the following equations: $3x_1 + 2x_2 = 18$, $x_1 = 4$, $x_2 = 6$, $x_1 = 0$ and $x_2 = 0$

Region represented by $3x_1 + 2x_2 \leq 18$

The line $3x_1 + 2x_2 = 18$ meets the coordinate axes at A(6, 0) and B(0, 9) respectively. By joining these points we obtain the line $3x_1 +$

$2x_2 = 18$

Clearly (0, 0) satisfies the inequation $3x_1 + 2x_2 = 18$. So, the region in the plane which contain the origin represents the solution set of the inequation $3x_1 + 2x_2 = 18$

Region represented by $x_1 \leq 4$:

The line $x_1 = 4$ is the line that passes through C(4, 0) and is parallel to the Y axis. The region to the left of the line $x_1 = 4$ will satisfy the inequation $x_1 \leq 4$

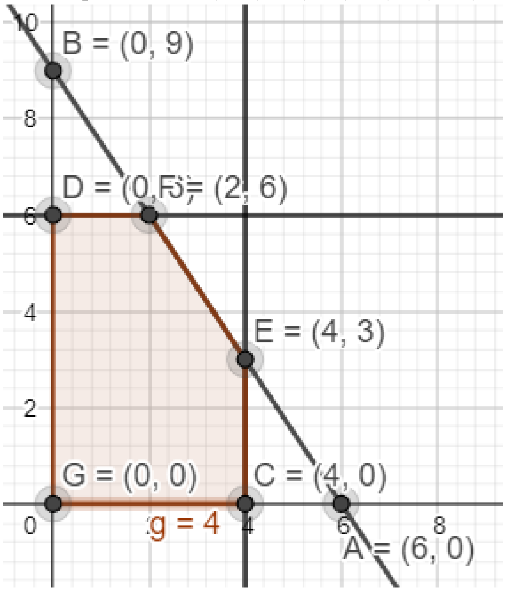
Region represented by $x_2 \leq 6$ The line $x_2 = 6$ is the line that passes through D(0, 6) and is parallel to the x axis. The region below the line $x_2 = 6$ will satisfy the inequation $x_2 \leq 6$.

Region represented by $x_1 \geq 0$ and $x_2 \geq 0$

since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations $x_1 \geq$ oand $x_2 \geq 0$

The feasible region determined by the system of constraints, $3x_1 + 2x_2 \leq 18$, $x_1 \leq 4$, $x_2 \leq 6$, $x_1 \geq 0$, and $x_2 \geq 0$, are as follows :

Corner points are O(0, 0), D(0, 6), F(2, 6), E(4, 3) and C(4, 0).



The values of the objective function at these points are given in the following table

Points : Value of Z

O(0, 0) : 3(0) + 5(0) = 0

D(0, 6) : 3(0) + 5(6) = 30

F(2, 6) : 3(2) + 5(6) = 36

E(4, 3) : 3(4) + 5(3) = 27

C(4, 0) : 3(4) + 5(0) = 12

We see that the maximum value of the objective function Z is 36 which is at F(2, 6)

36. **(a)** convex polygon

Explanation: Feasible region for an LPP is always a convex polygon.

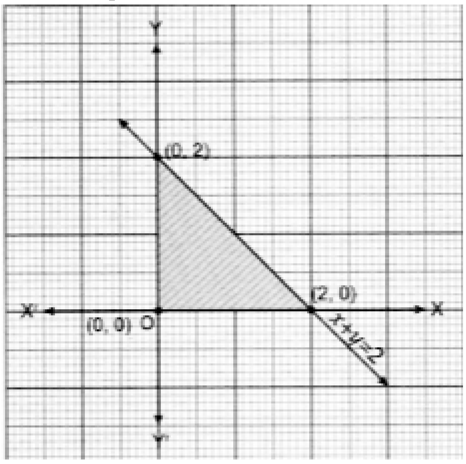
37. **(a)** 1020

Explanation: Here , Maximize Z = 5x+3y , subject to constraints x + y ≤ 300 , 2x + y ≤ 360, x ≥ 0, y ≥ 0.

Corner points	Z = 5x +3 y
P(0, 300)	900
Q(180, 0)	900
R(60, 240)	1020…………………..(Max.)
S(0, 0)	0

Hence, the maximum value is 1020

38. **(a)** at infinite number of points

Explanation: 

Feasible region is shaded region with corner points (0, 0), (2, 0) and (0, 2)

Z(0, 0) = 0

Z(2, 0) = 2⟵ maximise

Z(0, 2) = 2 ⟵ maximise

Z_{max} = 2 obtained at (2, 0) and (0, 2) so is obtained at any point on line segment joining (2, 0) and (0, 2).

39.

(c) R

Explanation:

Corner points	**Value of Z = 2x + 5y**
P(0, 5)	Z = 2(0) + 5(5) = 25
Q(1, 5)	Z = 2(1) + 5(5) = 27
R(4, 2)	Z = 2(4) + 5(2) = 18 → Minimum
S(12, 0)	Z = 2(12) + 5(0) = 24

Thus, minimum value of Z occurs ar R(4, 2)

40.

(b) an open half-plane containing the origin.

Explanation: The strict inequality represents an open half plane and it contains the origin as (0, 0) satisfies it.

41.

(d) 12

Explanation: Z = 2x + 3y

Z (3, 2) = 2 × 3 + 3 × 2

= 6 + 6 = 12

42.

(c) Linear constraints

Explanation: In a LPP, the linear inequalities or restrictions on the variables are called Linear constraints.

43. **(a)** 12

Explanation: The feasible region as shown in the figure, has objective function F = 3x - 4y.

Corner points	**Corresponding value of F = 3x - 4y**
(0, 0)	0
(12, 6)	12 ⟵ Maximum
(0, 4)	-16

Hence, the maximum value of F is 12.

44.

(b) Maximum value of Z is at Q.

Explanation:

Corner Points of Feasible Region	Value of Z = (Z = 4x + 3y)
O(0, 0)	Z = 4(0) + 3(0) = 0
P(0, 40)	Z = 4(0) + 3(40) = 120
Q(30, 20)	Z = 4(30) + 3(20) = 180 (Maximum)
R(40, 0)	Z = 4(40) + 3(0) = 160

Thus, Maximum value of Z is at Q, which is 180.

45.

(d) 47

Explanation:

Corner points	Z = 11x + 7y
(0, 5)	35
(0,3)	21
(3,2)	47

The maximum value is 47

46.

(b) - 17

Explanation:

Corner points	Z = 3x - 4y
(0, 0)	0
(5,0)	15(Max.)
(6,8)	-14
(6 , 5)	-2
(4,10)	-28
(0,8)	-32……………..(Min.)

Maximum value of Z + Minimum value of Z = 15 + (-32) = - 17 .

47.

(d) Maximum = 9, minimum = $3\frac{1}{7}$

Explanation:

Corner points	Z = x +2 y
P(3/13,24/13)	51/13
Q(3/2,15/4)	9…………………….(Max.)
R(7/2,3/4)	5
S(18/7,2/7)	22/7……………………(Min.)

Hence the maximum value is 9 and the minimum value is $3\frac{1}{7}$

48.

(d) 12

Explanation:

Corner points	Z = 3x - 4y
(0, 0)	0
(0,4)	-16
(12,6)	12…………………….(Max.)

49.

(d) (5, 0)

Explanation:

Corner points	Z = 3x - 4y
(0, 0)	0
(5,0)	15
(6,8)	-14
(6 , 5)	-2
(4,10)	-28
(0,8)	-32

The maximum value occurs at (5,0)

50.

(d) 43

Explanation:

Corner points	Z = 5x + 7y
O(0,0)	0
B (3,4)	43
A(7,0)	35
C(0,2)	14

Hence the maximum value is 43

51.

(d) 152

Explanation: Here , minimize Z = 3x + 4y ,

Corner points	Z = 3x + 4y
C(0, 38)	152……………………(Min.)
B (52, 0)	156
D(44, 16)	196

The minimum value is 152

52. **(a)** (0, 8)

Explanation:

Corner points	Z = 3x - 4y
(0, 0)	0
(5,0)	15

(6,8)	-14
(6 ,5)	-2
(4,10)	-28
(0,8)	-32(Min.)

The minimum value occurs at (0,8)

53.

(b) 21

Explanation:

Corner points	**Z = 11x + 7y**
(0, 5)	35
(0,3)	21
(3,2)	47

Hence the minimum value is 21

54.

(b) 196

Explanation: Here , maximize Z = 3x+4y ,

Corner points	**Z = 3x + 4 y**
C(0 ,38)	132
B (52 ,0)	156
D(44 , 16)	196

Hence the maximum value is 196

55. **(a)** Minimum value = 2

Explanation:

Corner points	**Z = 4x + y**
(0, 2)	2
(0,3)	3
(2,1)	9

Hence the minimum value is 2

56.

(b) 140 bags of brand P and 50 bags of brand Q; Maximum amount of nitrogen = 595 kg

Explanation: Let the number of bags used for fertilizer of brand P = x And the number of bags used for fertilizer of brand Q = y . Here , Z = 3x + 3.5y subject to constraints : :1.5 x +2 y ≤ 310, x + 2y ≥ 240, 3x + 1.5y ≥ 270 , x,y ≥ 0

Corner points	Z =3x + 3.5 y
C(40 ,100)	470……………….(Min.)
B (140,50)	595……………………..(Max.)
D(20,140)	550

Here Z = 595 is maximum i.e. 140 bags of brand P and 50 bags of brand Q; Maximum amount of nitrogen = 595 kg .