Solution

CET25M2 MATRICES

Class 12 - Mathematics

1.

(c) 8

Explanation: 8

2. **(a)** A + B

Explanation: AB = B ⇒ (AB)A = BA

⇒ A(BA) = BA ⇒ A(A) = A,

⇒ A^2 = A

AB = B ⇒ B(AB) = BB

⇒ (BA)B = B^2

⇒ AB = B^2

⇒ B = B^2

∴ $A^2 + B^2$ = A + B

3.

(b) 64

Explanation: The order of the matrix = 2 × 3

The number of elements = 2 × 3 = 6

Each place can have either 1 or 2. So, each place can be filled in 2 ways.

Thus, the number of possible matrices = 2^6 = 64

4. **(a)** (8, 8)

Explanation: A = $\begin{vmatrix} 3 & 1 \\ 7 & 5 \end{vmatrix}$, A^2 = $\begin{vmatrix} 3 & 1 \\ 7 & 5 \end{vmatrix}\begin{vmatrix} 3 & 1 \\ 7 & 5 \end{vmatrix} = \begin{vmatrix} 16 & 8 \\ 56 & 32 \end{vmatrix}$

∵ A^2 + xI = yA

$\Rightarrow \begin{vmatrix} 16 & 8 \\ 56 & 32 \end{vmatrix} + \begin{vmatrix} x & 0 \\ 0 & x \end{vmatrix} = \begin{vmatrix} 3y & y \\ 7y & 5y \end{vmatrix}$

$\Rightarrow \begin{vmatrix} 16+x & 8 \\ 56 & 32+x \end{vmatrix} = \begin{vmatrix} 3y & y \\ 7y & 5y \end{vmatrix}$

⇒ y = 8, 16 + x = 3y = 24 ⇒ x = 24 - 16 = 8

∴ x = 8, y = 8

5.

(d) $B^TA^T = (AB)^T$

Explanation: A = $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$, B = $\begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}$, A^T = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, B^T = $\begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}$

AB = $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 19 \\ 10 & 28 \end{bmatrix}$

$(AB)^T$ = $\begin{bmatrix} 7 & 19 \\ 10 & 28 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 19 & 28 \end{bmatrix}$

B^TA^T = $\begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 19 & 28 \end{bmatrix}$ = $(AB)^T$

6. **(a)** AB may or may not be defined.

Explanation: Let matrix A is of order m x n and matrix B is of order p x q. Then, the product AB is defined only when number of columns in A is equal to number of rows in B i.e n = p. Therefore, If A and B are any two matrices, then AB may or may not be defined.

7.

(d) Skew symmetric matrix

Explanation: Given A and B are symmetric matrix of same order

⇒ A = A' ....(i)

⇒ B = B' ......(ii)

So, AB - BA = A'B' - B'A' ...(from eqn (i) and (ii))

⇒ AB - BA = (BA)' - (AB)' ... (∵ (AB)' = B'A')

⇒ AB - BA = (-1) ((AB)' - (BA)')   ...(taking -1 common)

⇒ AB - BA = -(AB - BA)' ...(∵ (A - B)' = A' - B')

Here we see that the relation between (AB - BA) and its transpose i.e. (AB - BA)' is (AB - BA) = - (AB -BA)', this implies that (AB - BA) is a skew symmetric matrix.

8.

(b) square matrix

Explanation: square matrix, In a given matrix the number of rows is equal to the number of columns.

9.

(c) ± 5

Explanation: ± 5

10.

(b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Explanation: A = $\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$

$A^{4n} = A^4 = \begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix} \times \begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ i & 0 \end{bmatrix} \times \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$

{n = 1, so the exponent comes out to be 4 and if n = 2, which will turn the exponent to 8, and the same cycle will repeat.}

= $\begin{bmatrix} \mathrm{i}^4 & 0 \\ 0 & \mathrm{i}^4 \end{bmatrix}$

= $\begin{bmatrix} (-1)^2 & 0 \\ 0 & (-1)^2 \end{bmatrix}$

= $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

11. **(a)** $\begin{bmatrix} 2+\sqrt{3} & 1+\sqrt{5} & 0 \\ 0 & 6 & \frac{1}{2} \end{bmatrix}$

Explanation: Since, A, B are of the same order 2×3. Therefore, addition of A and B is defined and is given by

$A+B = \begin{bmatrix} 2+\sqrt{3} & 1+\sqrt{5} & 1-1 \\ 2-2 & 3+3 & 0+\frac{1}{2} \end{bmatrix}$

$= \begin{bmatrix} 2+\sqrt{3} & 1+\sqrt{5} & 0 \\ 0 & 6 & \frac{1}{2} \end{bmatrix}$

12.

(c) x = y

Explanation: $A = A^T$

$\begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix} = \begin{bmatrix} 5 & y \\ x & 0 \end{bmatrix}$

x = y

13.

(d) E (α + β)

Explanation: Given, E (θ) $= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

∴ E(α) E(β) = $\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}$

$= \begin{bmatrix} \cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta & \cos\alpha\cdot\sin\beta + \sin\alpha\cdot\cos\beta \\ -\sin\alpha\cdot\cos\beta - \sin\beta\cdot\cos\alpha & -\sin\alpha\cdot\sin\beta + \cos\alpha\cdot\cos\beta \end{bmatrix}$

$= \begin{bmatrix} \cos(\alpha+\beta) & \sin(\alpha+\beta) \\ -\sin(\alpha+\beta) & \cos(\alpha+\beta) \end{bmatrix} = E(\alpha+\beta)$

14.

(d) I

Explanation: Given that $A^2 = A$

Calculating value of $(I + A)^3 - 7A$:

$(I+A)^3 - 7A = I^3 + A^3 + 3I^2A + 3IA^2 - 7A$

$= \mathrm{I} + \mathrm{A}^2 \cdot \mathrm{A} + 3\mathrm{A} + 3\mathrm{A}^2 - 7\mathrm{A}\ (\mathrm{I}^n = \mathrm{I} \text{ and } \mathrm{I} \cdot \mathrm{A} = \mathrm{A})$

$= I + A \cdot A + 3A + 3A - 7A\left(A^2 = A\right)$

$= I + A^2 + 3A + 3A - 7A$

$= I + 7A - 7A$

Hence, $(I + A)^3 - 7A = I$

15.

(b) x = 2, y = 3

Explanation: We have, $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$

⇒ 4x = x + 6 ⇒ x = 2

and 4x = 7y - 13

⇒ 8 = 7y - 13

⇒ y = 3

16.

(d) A is a square matrix

Explanation: A square matrix is a null matrix if all its entries are zero.

17.

(c) 4

Explanation: Here, A = $\begin{bmatrix} 5 & 8 & 11 \\ 4 & 5 & 13 \\ 7 & 5 & 5 \end{bmatrix}$

Thus, number of elements more than 5, is 4.

18.

(c) $a_{ij} = 0$, where i = j

Explanation: In a skew-symmetric matrix, the (i, J)th element is negative of the (j, i)th element. Hence, the (i, i)th element = 0

19.

(b) 17

Explanation: As the trace of a matrix is the sum of all diagonal elements,

Therefore, 1 + 7 + 9 = 17

Trace = 17.

Which is the required solution.

20. **(a)** kA

Explanation: A = $[S_{ij}]$

S = $\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$

As, $S_{ij} = k$

Let A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ {Square Matrix}

AS = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \times \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$

= $\begin{bmatrix} \mathrm{ka}_{11} & \mathrm{ka}_{12} \\ \mathrm{ka}_{21} & \mathrm{ka}_{22} \end{bmatrix}$

= k $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

= kA

SA = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \times \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$

$= \begin{bmatrix} \mathrm{ka}_{11} & \mathrm{ka}_{12} \\ \mathrm{ka}_{21} & \mathrm{ka}_{22} \end{bmatrix}$

$= \mathrm{k} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

= kA

Hence, AS = SA = kA

21. **(a)** 64

Explanation: Number of all possible matrix = $(2)^6$ = 64

22.

(d) $\theta = 2n\pi + \frac{\pi}{3}, n \in Z$

Explanation: A = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$A^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$A + A^T = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} + \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

2Cos θ = 1

Cos $\theta = \frac{1}{2}$

θ = 2n$\pi + \frac{\pi}{3}$ {n $\in$ Z}

23.

(c) $(AB)^T = B^T A^T$

Explanation: A = $\begin{vmatrix} -2 \\ 4 \\ 5 \end{vmatrix}$, B = [1 3 -6]

AB = $\begin{vmatrix} -2 \\ 4 \\ 5 \end{vmatrix}$ [1 3 -6] = $\begin{vmatrix} -2 & -6 & 12 \\ 4 & 12 & -24 \\ 5 & 15 & -30 \end{vmatrix}$

$B^T = \begin{vmatrix} 1 \\ 3 \\ -6 \end{vmatrix}$, A^T = [-2 4 5]

$B^T A^T = \begin{vmatrix} 1 \\ 3 \\ -6 \end{vmatrix}$ [-2 4 5] = $\begin{vmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ 12 & -24 & -30 \end{vmatrix}$ = $(AB)^T$

Also $(AB)^T = A^T B^T$

O(AB) = 3 $\times$ 3 and O(BA) = 1 $\times$ 1

$\therefore$ O(AB) $\neq$ O(BA)

24. **(a)** x = 2

Explanation: Given, $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$

We know that, if A is a skew-symmetric matrix, then

A = $-A^T$ ...(i)

From Eq. (i) We, get

$\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix} = - \begin{bmatrix} 0 & -1 & x \\ 1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -x \\ -1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}$

On comparing the corresponding element, we get

-2 = -x $\Rightarrow$ x = 2

25. **(a)** 1

Explanation: 1

26.

(d) 5I

Explanation: $A^2 = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$

$= \begin{pmatrix} 9 & 16 \\ 8 & 17 \end{pmatrix}$

$A^2 - 4A = \begin{pmatrix} 9 & 16 \\ 8 & 17 \end{pmatrix} - 4\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$

$= \begin{pmatrix} 9 & 16 \\ 8 & 17 \end{pmatrix} - \begin{pmatrix} 4 & 16 \\ 8 & 12 \end{pmatrix}$

$= \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$

$= 5\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

= 5I

27.

(c) $3 \times$ n

Explanation: $A_{3 \times m}$ and $B_{3 \times n}$ are two matrices. If m = n then A and B same orders as $3 \times$ n each so the order of (5A - 2B) should be same as $3 \times$ n.

28.

(b) $a_{ij} = b_{ij}$

Explanation: Two matrices A = $[a_{ij}]$ and B = $[b_{ij}]$ are said to be equal, if

i. they are of the same order.
ii. each element of A is equal to the corresponding element of B i.e. $a_{ij} = b_{ij}$ by for all i and j.

29.

(b) a = 1, b = 4

Explanation: A + B= $\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$

= $\begin{bmatrix} a+1 & 0 \\ b+2 & -2 \end{bmatrix}$

$(A + B)^2$ = $\begin{bmatrix} a+1 & 0 \\ b+2 & -2 \end{bmatrix} \times \begin{bmatrix} a+1 & 0 \\ b+2 & -2 \end{bmatrix}$

= $\begin{bmatrix} (a+1)^2+0 & 0+0 \\ (b+2)(a+1)-4-b & 0+4 \end{bmatrix}$

= $\begin{bmatrix} a^2+2a+1 & 0 \\ 2+2a+b+ab-4-b & 4 \end{bmatrix}$

= $\begin{bmatrix} a^2+2a+1 & 0 \\ 2a+ab-2 & 4 \end{bmatrix}$

A^2 = $\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} \times \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$

A^2 = $\begin{bmatrix} 1-2 & -1+1 \\ 2-2 & -2+1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

B^2 = $\begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} \times \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$

B^2 = $\begin{bmatrix} a^2+b & a^2-1 \\ ab-b & b+1 \end{bmatrix}$

$A^2 + B^2$ = $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} a^2+b & a^2-1 \\ ab-b & b+1 \end{bmatrix}$

$A^2 + B^2$ = $\begin{bmatrix} a^2+b-1 & a^2-1 \\ ab-b & b \end{bmatrix}$

As, $A^2 + B^2 = (A+B)^2$

$\therefore \begin{bmatrix} a^2+b-1 & a^2-1 \\ ab-b & b \end{bmatrix} = \begin{bmatrix} a^2+2a+1 & 0 \\ 2+2a+b+ab-4-b & 4 \end{bmatrix}$

$a^2 = 1$ and b = 4

a = $\pm$ 1 and b = 4

30. **(a)** $\begin{bmatrix} 7 & -5 \\ 1 & 5 \end{bmatrix}$

Explanation: Given that,

$(A+B) = \begin{bmatrix} 4 & -3 \\ 1 & 6 \end{bmatrix}$ ...(i)

$(A-B) = \begin{bmatrix} -2 & -1 \\ 5 & 2 \end{bmatrix}$ ...(ii)

$(i) + (ii) \Rightarrow 2A = \begin{bmatrix} 4 & -3 \\ 1 & 6 \end{bmatrix} + \begin{bmatrix} -2 & -1 \\ 5 & 2 \end{bmatrix}$

$\Rightarrow 2A = \begin{bmatrix} 2 & -4 \\ 6 & 8 \end{bmatrix}$

Dividing the matrix by 2

$\Rightarrow A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$

$(i) - (ii) \Rightarrow 2B = \begin{bmatrix} 4 & -3 \\ 1 & 6 \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 5 & 2 \end{bmatrix}$

$\Rightarrow$ 2B = $\begin{bmatrix} 6 & -2 \\ -4 & 4 \end{bmatrix}$

Dividing the matrix by 2

$\Rightarrow$ B = $\begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$

$A \times B = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$

$= \begin{bmatrix} 1 \times 3 + (-2) \times (-2) & (1) \times (-1) + (-2) \times (2) \\ 3 \times 3 + 4 \times (-2) & 3 \times (-1) + 4 \times 2 \end{bmatrix}$

$= \begin{bmatrix} 7 & -5 \\ 1 & 5 \end{bmatrix}$

31.

(d) A is non-singular matrix

Explanation: Here, only non- singular matrices obey cancellation laws.

32.

(b) $\begin{vmatrix} -3 & 4 & -1 \\ -3 & 0 & -1 \end{vmatrix}$

Explanation: A = $\begin{vmatrix} 1 & -3 & 2 \\ 2 & 0 & 2 \end{vmatrix}$, B = $\begin{vmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \end{vmatrix}$

A + B = $\begin{vmatrix} 3 & -4 & 1 \\ 3 & 0 & 1 \end{vmatrix}$

$\because$ A + B + C = 0 $\Rightarrow$ C = -(A + B) = $\begin{vmatrix} 3 & -4 & 1 \\ 3 & 0 & 1 \end{vmatrix}$

$\therefore$ C = $\begin{vmatrix} -3 & 4 & -1 \\ -3 & 0 & -1 \end{vmatrix}$

33. **(a)** -2

Explanation: Now,

$A^2 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1+1 & -1-1 \\ -1-1 & 1+1 \end{bmatrix}$

$= \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$

$A^2 = 2 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

Given A^2 = kA

$\therefore 2 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = k \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$\Rightarrow$ k = 2

34.

(c) $\begin{bmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{bmatrix}$

Explanation: $4A - 2B = \begin{pmatrix} 12 & -12 & 0 \\ -8 & 4 & 2 \end{pmatrix}$ ...(i)

$2B + A = \begin{pmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{pmatrix}$ ...(ii)

(i) + (ii)

$5A = \begin{pmatrix} 12 & -12 & 0 \\ -8 & 4 & 2 \end{pmatrix} + \begin{pmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{pmatrix}$

$= \begin{pmatrix} 15 & -10 & 5 \\ -10 & 5 & -5 \end{pmatrix}$

Dividing each elements of the matrix by 5

$A = \begin{bmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{bmatrix}$

35.

(c) $A^2 = A$

Explanation: $A^2 = \begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix}\begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} = A$

36.

(c) $\frac{1}{3}\begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix}$

Explanation: $A + B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ ...(i)

$A - 2B = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$ ...(ii)

adding $2 \times$ (i) and (ii), we get

$2A + 2B = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix}$ ...(iii)

$A - 2B = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$ ...(iv)

adding (iii) and (iv), we get

$\Rightarrow 3A = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$

$\Rightarrow A = \frac{1}{3}\begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix}$

37. **(a)** $1 - \alpha^2 - \beta\gamma = 0$

Explanation: In the given question: $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$

Calculating A^2: $A^2 = A.A$

$= \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$

$= \begin{bmatrix} \alpha \cdot \alpha + \beta \cdot \gamma & \alpha \cdot \beta + \beta \cdot (-\alpha) \\ \gamma \cdot \alpha + (-\alpha) \cdot \gamma & \gamma \cdot \beta + (-\alpha) \cdot (-\alpha) \end{bmatrix}$

$= \begin{bmatrix} \alpha^2 + \beta\gamma & \alpha \cdot \beta - \alpha \cdot \beta \\ \gamma \cdot \alpha - \gamma \cdot \alpha & \gamma \cdot \beta + \alpha^2 \end{bmatrix}$

$= \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \beta\gamma + \alpha^2 \end{bmatrix}$

And given that $A^2 = I$

Therefore, $\begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \beta\gamma + \alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Comparing corresponding elements we obtain

$\Rightarrow \alpha^2 + \beta\gamma = 1$

$\Rightarrow 1 - \alpha^2 - \beta\gamma = 0$

38.

(c) pI

Explanation: A = $\begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{vmatrix}$ A^2 = $\begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{vmatrix}\begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1 \\ p & q & r \\ pr & p+qr & q+r^2 \end{vmatrix}$

A^3 = $\begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{vmatrix}\begin{vmatrix} 0 & 0 & 1 \\ p & q & r \\ pr & p+qr & q+r^2 \end{vmatrix}$ = $\begin{vmatrix} p & q & r \\ pr & p+qr & q+r^2 \\ pq+pr^2 & q^2+pr+qr^2 & p+qr+qr+r^3 \end{vmatrix}$

A^3 - rA^2 - qA = $\begin{vmatrix} p & q & r \\ pr & p+qr & q+r^2 \\ pq+pr^2 & q^2+pr+qr^2 & p+2qr+r^3 \end{vmatrix}$ + $\begin{vmatrix} 0 & 0 & -r \\ -pr & -qr & -r^2 \\ -pr^2 & -pr-qr^2 & -rq-r^3 \end{vmatrix}$ + $\begin{vmatrix} 0 & -q & 0 \\ 0 & 0 & -q \\ -pq & -q^2 & -qr \end{vmatrix}$

= $\begin{vmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{vmatrix}$ = pI

39. **(a)** A^2 = 0

Explanation: ∵ A = $\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}$,

A^2 = $\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix}$ = 0

∴ A is a null matrix.

40.

(b) AB $\neq$ BA

Explanation: Here, order of matrix A is 2×3 and order of matrix B is 3×2 So, AB will be of order 2×2 and BA will be of order 3×3.

Now, $AB = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2-8+6 & 3-10+3 \\ -8+8+10 & -12+10+5 \end{bmatrix}$ [multiplying rows by columns]

$AB = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix}$ and $BA = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$

$= \begin{bmatrix} 2-12 & -4+6 & 6+15 \\ 4-20 & -8+10 & 12+25 \\ 2-4 & -4+2 & 6+5 \end{bmatrix}$ [multiplying rows by columns]

$= \begin{bmatrix} -10 & 2 & 21 \\ -16 & 2 & 37 \\ -2 & -2 & 11 \end{bmatrix}$

AB $\neq$ BA

41. **(a)** $\begin{bmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{bmatrix}$

Explanation: $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

$= \begin{bmatrix} a \times a + b \times b & a \times (-b) + b \times a \\ (-b) \times a + a \times b & (-b) \times (-b) + a \times a \end{bmatrix}$

$= \begin{bmatrix} a^2+b^2 & -ab+ab \\ -ab+ab & b^2+a^2 \end{bmatrix}$ = $\begin{bmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{bmatrix}$

42.

(c) a_{32}

Explanation: The given matrix is A $= \begin{bmatrix} 4 & -2 & 1 & 3 \\ 5 & 7 & 9 & 6 \\ 21 & 15 & 18 & -25 \end{bmatrix}$

Here, a_{23} = 9 and a_{24} = 6

$\therefore a_{23} + a_{24} = 9 + 6 = 15$

Also, 15 lies in 3rd row and 2nd column.

$\therefore 15 = a_{32}$

43. **(a)** A - A^T is a skew symmetric matrix

Explanation: $(A - A^T)^T = A^T - (A^T) = A^T - A$

$= -(A - A^T)^T$

$\therefore$ A - A^T is a skew symmetric matrix.

44.

(d) $\begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix}$

Explanation: Given, $A_\alpha = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$

$A_a^2 = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$

$= \begin{pmatrix} \cos\alpha \times \cos\alpha - \sin\alpha \times \sin\alpha & \cos\alpha \times \sin\alpha + \sin\alpha \times \cos\alpha \\ -\sin\alpha \times \cos\alpha - \cos\alpha \times \sin\alpha & -\sin\alpha \times \sin\alpha + \cos\alpha \times \cos\alpha \end{pmatrix}$

$= \begin{pmatrix} \cos^2\alpha - \sin^2\alpha & \cos\alpha\sin\alpha + \sin\alpha\cos\alpha \\ -\sin\alpha\cos\alpha - \cos\alpha\sin\alpha & -\sin^2\alpha + \cos^2\alpha \end{pmatrix}$

$= \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{pmatrix}$

45.

(d) 7

Explanation: $\begin{bmatrix} \cos\frac{2\pi}{7} & -\sin\frac{2\pi}{7} \\ \sin\frac{2\pi}{7} & \cos\frac{2\pi}{7} \end{bmatrix}^k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$|A| = \cos^2\frac{2\pi}{7} - \sin\frac{2\pi}{7}\left(-\sin\frac{2\pi}{7}\right)$

$= \text{Cos}^2\frac{2\pi}{7} + \text{Sin}^2\frac{2\pi}{7}$

I = 1

$I^k = I$ {K can be anything}

Let $\theta = \frac{2\pi}{7}$

$A^2 = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \times \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$= \begin{bmatrix} \cos^2\theta - \sin^2\theta & -\sin\theta\cos\theta - \sin\theta\cos\theta \\ \sin\theta\cos\theta + \sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}$

As $\{\cos^2\theta - \sin^2\theta = \cos^2\theta$ and $2\text{Sin}\,\theta\cos\theta = \sin 2\theta\}$

$= \begin{bmatrix} \cos 2\theta & -2\sin\theta\cos\theta \\ 2\sin\theta\cos\theta & \cos 2\theta \end{bmatrix}$

$= \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$

$A^4 = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix} \times \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$

$= \begin{bmatrix} \cos 4\theta & -\sin 4\theta \\ \sin 4\theta & \cos 4\theta \end{bmatrix}$

Similarly, $A^7 = \begin{bmatrix} \cos 7\theta & -\sin 7\theta \\ \sin 7\theta & \cos 7\theta \end{bmatrix}$

Hence, $\theta = \frac{2\pi}{7}$

$7\theta = 2\pi$

Multiplying Cos & Sin, to LHS & RHS,

$\text{Cos}7\theta = \text{Cos}2\pi = 1$

$\text{Sin}7\theta = \sin 2\theta = 0$

$\begin{bmatrix} \cos 7\theta & -\sin 7\theta \\ \sin 7\theta & \cos 7\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

So, k = 7

$A^7 = I$

Hence, k = 7.

46. **(a)** $\begin{bmatrix} 2 & 6 \\ 10 & 4 \end{bmatrix}$

Explanation: Given, $A = \begin{bmatrix} 1 & 3 \\ 5 & 2 \end{bmatrix}$

$\therefore 2A = 2\begin{bmatrix} 1 & 3 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 10 & 4 \end{bmatrix}$

47.

(d) 1

Explanation: 1

48. **(a)** $\begin{bmatrix} 4 & 1 & 4 \\ 3 & -2 & 5 \\ 1 & 3 & -1 \end{bmatrix}$

Explanation: Let $A = \begin{bmatrix} 4 & 3 & 1 \\ 1 & -2 & 3 \\ 4 & 5 & -1 \end{bmatrix}$

Then, $A' = \begin{bmatrix} 4 & 3 & 1 \\ 1 & -2 & 3 \\ 4 & 5 & -1 \end{bmatrix}' = \begin{bmatrix} 4 & 1 & 4 \\ 3 & -2 & 5 \\ 1 & 3 & -1 \end{bmatrix}$ [interchanging the elements of rows and columns]

49. **(a)** $\begin{vmatrix} p^5 & q\left(\frac{p^5-1}{p-1}\right) \\ 0 & 1 \end{vmatrix}$

Explanation: $A^2 = \begin{vmatrix} p & q \\ 0 & 1 \end{vmatrix}\begin{vmatrix} p & q \\ 0 & 1 \end{vmatrix}$

$= \begin{vmatrix} p^2 & pq+q \\ 0 & 1 \end{vmatrix} = \begin{vmatrix} p^2 & q(p+1) \\ 0 & 1 \end{vmatrix}$

$A^3 = \begin{vmatrix} p^3 & q(p^2+p+1) \\ 0 & 1 \end{vmatrix}$

Similarly $A^4 = \begin{vmatrix} p^4 & q(p^3+p+1) \\ 0 & 1 \end{vmatrix}$

And $A^5 = \begin{vmatrix} p^5 & q(p^4+p^2+p+1) \\ 0 & 1 \end{vmatrix}$

$\Rightarrow A^5 = \begin{vmatrix} p^5 & q\left(\frac{p^5-1}{p-1}\right) \\ 0 & 1 \end{vmatrix}$ [$\because p^4 + p^3 + p^2 + p + 1$ is in G.P. with a = 1 and r = p]

50.

(c) $\begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a^n \end{bmatrix}$

Explanation: $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$

$A^n = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix} \times \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix} \times \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix} \times \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$ ....{n times, (where $n \in N$)}

$A^n = \begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a^n \end{bmatrix}$

51.

(c) A skew-symmetric matrix

Explanation: Let X = A - A'

X' = (A - A')'

= A' - (A')'

=A’ - A

= -(A - A’)

= -X

Therefore (A - A’) is skew symmetric matrix.

52.

(b) $A^2 – B^2$ + BA – AB

Explanation: (A + B) (A – B) = A (A – B) + B (A – B) = A^2 – AB + BA – B^2

53.

(d) A and B are square matrices of same order

Explanation: We know that if A and B are of the matrix of same order then both operations A + B and AB are well defined.

54.

(b) A is a zero matrix

Explanation: Only a null matrix can be symmetric as well as skew symmetric.

In Symmetric Matrix A^T = A,

Skew Symmetric Matrix A^T = -A,

Given that the matrix is satisfying both the properties.

Therefore, Equating the RHS we get A = -A i.e, 2A = 0.

Therefore A = 0, which is a null matrix.

55.

(b) m × n

Explanation: Let A = $[a_{ij}]_{m \times n}$ and B = $[b_{ij}]_{p \times q}$

∴ B' = $[b_{ji}]_{q \times p}$

Now, AB' is defined, so n = q

and B' A is also defined, so p = m

∴ order of B is m × n

56.

(b) A^T = A

Explanation: Since transpose of a symmetric matrix is equal to the matrix itself so, for a symmetric matrix A^T = A

57.

(d) 2 × n

Explanation: Matrix X is of the order 2 × n.

Therefore, matrix 7X is also of the same order.

Matrix Z is of order 2 × p $= 2 \times n \quad \ldots (\because p = n)$

⇒ Matrix 5Z is also of the same order.

Now, both the matrices 7X and 5Z are of the order 2 × n.

Thus, matrix 7X – 5Z is well- defined and is of the order 2 × n.

58.

(b) 0

Explanation: AB = $\begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -2 & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

= $\begin{bmatrix} 1+0+0 & -2+2+0 & y+0+x \\ 0+0+0 & 0+1+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$

= $\begin{bmatrix} 1 & 0 & x+y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

I_3 = $\begin{bmatrix} 1 & 0 & \mathrm{x+y} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Hence x + y = 0

59. **(a)** 64

Explanation: 64

60.

(d) AB and BA both are defined

Explanation: In given matrix

order of A = 2×3

order of B $= 3 \times 2$

AB will be defined if the number of column in A is equal to the number of rows in B

so, $(A_{2\times3})(B_{3\times2}) = AB_{2\times2}$

Similarly $(B_{3\times2})(A_{2\times3}) = BA_{3\times3}$

Thus, Both AB and BA are defined.

61.

(d) (x = 2, y = -8)

Explanation: $2\begin{pmatrix} 3 & 4 \\ 5 & x \end{pmatrix} + \begin{pmatrix} 1 & y \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 7 & 0 \\ 10 & 5 \end{pmatrix}$

To solve this problem we will use the comparison that is we will use that all the elements of L.H.S. are equal to R.H.S.

$= \begin{pmatrix} 6 & 8 \\ 10 & 2x \end{pmatrix} + \begin{pmatrix} 1 & y \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} 7 & 8+y \\ 10 & 2x+1 \end{pmatrix}$

Comparing with R.H.S.

8 + y = 0

y = -8

2x + 1 = 5

2x = 4

x = 2

62.

(b) -6

Explanation: -6

63.

(b) I = $[a_{ij}]_n$, where a_{ij} = $\begin{cases} 1 \text{ if } i = j \\ 0 \text{ if } i \neq j \end{cases}$

Explanation: By definition of identity matrix

I = $[a_{ij}]_n$, where a_{ij} = $\begin{cases} 1 \text{ if } i = j \\ 0 \text{ if } i \neq j \end{cases}$

64. **(a)** $[a_{ji}]_{n\times m}$

Explanation: If $A = [a_{ij}]_{m\times n}$, then A' $= [a_{ji}]_{m\times n}$.

65.

(c) $A^2 + AB + BA + B^2$

Explanation: Since A and B are square matrices of same order.

$(A + B)^2 = (A + B)(A + B)$

$= A^2 + AB + BA + B^2$

66.

(b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Explanation: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

67. **(a)** A is a zero matrix

Explanation: If a matrix A is both symmetric and skew-symmetric,

A’ = A & A’ = -A

Comparing both the equations,

A = -A

A + A = 0
2A = 0
A = 0
then A is a zero matrix.

68. **(a)** 40
Explanation: 40

69.
(b) 3 × 3
Explanation: $\begin{bmatrix}1 & -1\\0 & 2\\2 & 3\end{bmatrix}_{3\times2}\left\{\begin{bmatrix}-1 & 0 & 2\\2 & 0 & 1\end{bmatrix}_{2\times3}-\begin{bmatrix}0 & 1 & 23\\1 & 0 & 21\end{bmatrix}_{2\times3}\right\}$
$=\begin{bmatrix}1 & -1\\0 & 2\\2 & 3\end{bmatrix}_{3\times2}\begin{bmatrix}-1 & -1 & -21\\1 & 0 & -20\end{bmatrix}_{2\times3}$
$=\begin{bmatrix}-2 & -1 & -1\\2 & 0 & -40\\1 & -2 & -102\end{bmatrix}_{3\times3}$

70. **(a)** a symmetric matrix
Explanation: Symmetric matrix. Since, A' = A, therefore, $\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}=\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$

71.
(c) Nilpotent
Explanation: The given matrix A is nilpotent, because |A| = 0, as determinant of a nilpotent matrix is zero.

72.
(b) $\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}$
Explanation: The orthogonal projection on x-axis is given by: $\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}$.

73.
(c) -7
Explanation: $\because A=\begin{bmatrix}1 & 0\\-1 & 7\end{bmatrix}, I=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$
Now, $A^2=\begin{bmatrix}1 & 0\\-1 & 7\end{bmatrix}\begin{bmatrix}1 & 0\\-1 & 7\end{bmatrix}=\begin{bmatrix}1 & 0\\-8 & 49\end{bmatrix}$ and 8A + KI $=\begin{bmatrix}8 & 0\\-8 & 56\end{bmatrix}+\begin{bmatrix}k & 0\\0 & k\end{bmatrix}$
$=\begin{bmatrix}8+k & 0\\-8 & 56+k\end{bmatrix}$
$\because A^2 = 8A + KI$
$\therefore \begin{bmatrix}1 & 0\\-8 & 49\end{bmatrix}=\begin{bmatrix}8+k & 0\\-8 & 56+k\end{bmatrix}$
$\Rightarrow$ 8 + k = 1
$\Rightarrow$ k = -7

74.
(b) AB = BA = I
Explanation: Here it is given that A & B are inverse of each other.
$\therefore A^{-1} = B$ ...(i)
Also $B^{-1} = A$ ...(ii)
From definition of inverse matrix, we know that-
$AA^{-1} = I$
$\therefore A^{-1} = B$ ...from eq (i)
$\Rightarrow AB = AA^{-1}$ = I...(iii)
Similarly, $BB^{-1} = I$

$\therefore B^{-1} = A$ ...(from eq(ii))

$\Rightarrow BA = BB^{-1} = I$ ...(iv)

So, from (iii) and (iv), we get

AB = BA = I

75. **(a)** $(2n + 1)\frac{\pi}{2}$

Explanation: Now $A \cdot B = \begin{vmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{vmatrix} \cdot \begin{vmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{vmatrix}$

$= \begin{vmatrix} \cos^2\theta\cos^2\phi + \cos\theta\cos\phi\sin\theta\sin\phi & \cos^2\theta\cos\phi\sin\phi + \sin^2\phi\sin\theta\cos\theta \\ \cos^2\phi\cos\theta\sin\theta + \sin^2\theta\cos\phi\sin\phi & \cos\theta\sin\theta\cos\phi\sin\phi + \sin^2\theta\sin^2\phi \end{vmatrix}$

$= \begin{vmatrix} \cos\theta\cos\phi\cos(\theta-\phi) & \cos\theta\sin\phi\cos(\theta-\phi) \\ \cos\phi\sin\theta\cos(\theta-\phi) & \sin\theta\sin\phi\cos(\theta-\phi) \end{vmatrix}$

From question $A \cdot B = 0$ (zero matrix)

$\Rightarrow \begin{vmatrix} \cos\theta\cos\phi\cos(\theta-\phi) & \cos\theta\sin\phi\cos(\theta-\phi) \\ \cos\phi\sin\theta\cos(\theta-\phi) & \sin\theta\sin\phi\cos(\theta-\phi) \end{vmatrix} = 0$

$\Rightarrow \cos(\theta-\phi)\begin{vmatrix} \cos\theta\cos\phi & \cos\theta\sin\phi \\ \cos\phi\sin\theta & \sin\theta\sin\phi \end{vmatrix} = 0$

$\Rightarrow \cos(\theta-\phi) = 0$

$\Rightarrow \theta - \phi = (2n+1)\frac{\pi}{2}$