ABHINAV ACADEMY

UDUPI

CET25M5 CONTINUITY AND DIFFERENTIABILITY

Class 12 - Mathematics

Time Allowed: 1 hour and 30 minutes **Maximum Marks: 75**

1. If f(x) = |x - a| ϕ (x), where ϕ (x) is continuous function, then **[1]**

   a) $f'(a^+) \neq f'(a^-)$

   b) $f'(a^-) = +\phi(a)$

   c) $f'(a^+) = \phi(a)$

   d) $f'(a^+) = f'(a^-)$

2. All the points of discontinuity of the function f defined by f(x) = $\begin{cases} 3, & \text{if } 0 \leq x \leq 1 \\ 4, & \text{if } 1 < x < 3 \\ 5, & \text{if } 3 \leq x \leq 10 \end{cases}$ are **[1]**

   a) 1, 3, 10

   b) 0, 1, 3

   c) 3, 10

   d) 1, 3

3. The function f(x) = $\begin{cases} 2-x, & \text{if } x < 2 \\ 2+x, & \text{if } x \geq 2 \end{cases}$ at x = 2 **[1]**

   a) is continuous

   b) has jump discontinuity

   c) has removable discontinuity

   d) has oscillating discontinuity

4. The function $f(x) = e^{|x|}$ **[1]**

   a) differentiable everywhere except at x = 0

   b) differentiable at x = 0

   c) differentiable everywhere

   d) differentiable at x = -1

5. If $y = \tan^{-1} x + \cot^{-1} x + \sec^{-1} x + \text{cosec}^{-1} x$, then $\frac{dy}{dx}$ is equal to **[1]**

   a) π

   b) 1

   c) $\frac{x^2-1}{x^2+1}$

   d) 0

6. Let f(x) = |x| and g(x) = $|x^3|$, then **[1]**

   a) f(x) and g(x) both are differentiable at x = 0

   b) f(x) is differentiable but g(x) is not differentiable at x = 0

   c) f(x) and g(x) both are continuous at x = 0

   d) f(x) and g(x) both are not differentiable at x = 0

7. Find the value of f(0), so that the function $f(x) = \frac{\sqrt{a^2-ax+x^2}-\sqrt{a^2+ax+x^2}}{\sqrt{a+x}-\sqrt{a-x}}$ becomes continuous for all x, given by **[1]**

   a) $a^{1/2}$

   b) $-a^{1/2}$

   c) $a^{3/2}$

   d) $-a^{3/2}$

8. If $y = \frac{\cos x - \sin x}{\cos x + \sin x}$, then $\frac{dy}{dx}$ is: **[1]**

a) -sec$^2\left(\frac{\pi}{4}-x\right)$ b) sec$^2\left(\frac{\pi}{4}-x\right)$

c) $-\log\left|\sec\left(\frac{\pi}{4}-x\right)\right|$ d) $\log\left|\sec\left(\frac{\pi}{4}-x\right)\right|$

9. If $y=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$, then $\frac{dy}{dx}$ is equal to **[1]**

a) $\frac{2}{1+x^2}$ b) $\frac{-2}{1+x^2}$

c) $\frac{1}{1+x^2}$ d) $\frac{2}{1-x^2}$

10. If x = a cos$^3\theta$, y = a sin$^3\theta$, then $\sqrt{1+\left(\frac{dy}{dx}\right)^2}=$ **[1]**

a) sec θ b) tan$^2\theta$

c) sec$^2\theta$ d) |sec θ|

11. If y = x^x find $\frac{d^2y}{dx^2}$ **[1]**

a) 0 b) x^x{(1 - log x)^2 + $\frac{1}{x}$}

c) x^x{(1 + log x)^2 + $\frac{1}{x}$} d) x^x{(1 + log x)^2 - $\frac{1}{x}$}

12. If y = x$^{x\sin x}$ then $\frac{dy}{dx}$ = ? **[1]**

a) $x^{\sin x}\left\{\frac{\sin x+x\log x\cdot\sin x}{x}\right\}$ b) $(\sin x\cos x)\cdot x^{(\sin x-1)}$

c) $2x^{\sin x}\left\{\frac{\sin x-x\log x\cdot\sin x}{x}\right\}$ d) $(\sin x)\cdot x^{(\sin x-1)}$

13. The function f(x) = $\begin{cases}\frac{\sin x}{x}+\cos x, & \text{if } x\neq 0\\ 1, & \text{if } x=0\end{cases}$ at x = 0 **[1]**

a) is continuous b) has removable discontinuity

c) has oscillating discontinuity d) has jump discontinuity

14. If x = acos^{3}t, y = asin^{3}t, then $\frac{dy}{dx}$ is equal to **[1]**

a) – tan t b) cosec t

c) cos t d) cot t

15. If f(x) = x^n, then the value of $f(1)+\frac{f(1)}{1!}+\frac{f^2(1)}{2!}+\frac{f^3(1)}{3!}+\ldots+\frac{f^n(1)}{n!}$, where f^r(1) is the rth derivative of f(x) w.r.t. x. **[1]**

a) 2^n b) n^2

c) 1 d) n

16. If $\sin^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right)=\log a$ then $\frac{dy}{dx}$ is equal to **[1]**

a) $\frac{x^2-y^2}{x^2+y^2}$ b) $\frac{y}{x}$

c) $\frac{x}{y}$ d) $\frac{x^2+y^2}{x^2-y^2}$

17. If $f(x)=\frac{1}{1-x}$, then the set of points of discontinuity of the function $f(f(f(x)))$ is **[1]**

a) {-1, 1} b) {1, 1}

c) {1} d) {0, 1}

18. The function $f(x)=\begin{cases}\frac{e^{3x}-e^{-5x}}{x}, & \text{if } x\neq 0\\ k & \text{if } x=0\end{cases}$ is continuous at x = 0 for the value of k, as: **[1]**

a) 8 b) 3

c) 5 d) 6

19. Let f(x) and g(x) be twice differentiable functions on [0, 2] satisfying f''(x) = g''(x), f'(1) = 4, g'(1) = 6, f(2) = 3 and g(2) = 9. Then, what is f(x) - g(x) at x = 4 equal to? **[1]**

a) -4 b) -10

c) 2 d) -6

20. The function f(x) = x – [x], where [.] denotes the greatest integer function is **[1]**

a) continuous at non-integer points only b) continuous everywhere

c) continuous at integer points only d) differentiable everywhere

21. If $y = e^{\tan x}$, then $(\cos^2 x)\, y_2$ = **[1]**

a) $(1 + \sin 2x)\, y_1$ b) $(1 + \sin 4x)\, y_1$

c) $(1 - \sin 2x)\, y_1$ d) $-(1 + \sin 2x)\, y_1$

22. The derivative of $\log(\cos e^x)$ is **[1]**

a) $-e^x \tan(e^x)$ b) $e^x \tan e^x$

c) $-\tan(e^x)$ d) $-\tan(e) - \tan(e^x)$

23. If $y = ax^2 + bx + c$, then $y^3 \frac{d^2y}{dx^2}$ is **[1]**

a) a constant b) a function of x only

c) a function of y only d) a function of x and y

24. Let f(x) = $\begin{cases} \cos[x], & x \geq 0 \\ |x| + a, & x < 0 \end{cases}$ where [x] denotes the greatest integer $\leq$ x. If $\lim_{x \to 0}$ f(x) exists then a is equal to **[1]**

a) 3 b) 0

c) -1 d) 1

25. If $y = \tan^{-1}\left(\frac{1+x^2}{1-x^2}\right)$ then $\frac{dy}{dx} =$? **[1]**

a) $\frac{-2x}{(1+x^4)}$ b) $\frac{2x}{(1-x^4)}$

c) $\frac{2x}{(1+x^4)}$ d) $\frac{x}{(1+x^4)}$

26. If x cos y + y cos x = π then y'(0) is **[1]**

a) 1 b) π

c) 0 d) $-\pi$

27. If $f(x) = \begin{cases} mx + 1, & \text{if } x \leq \frac{\pi}{2} \\ \sin x + n, & \text{if } x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$ then **[1]**

a) $m = n = \frac{\pi}{2}$ b) $n = \frac{m\pi}{2}$

c) m = 1, n = 0 d) $m = \frac{n\pi}{2} + 1$

28. If $x^y = y^x$, find $\frac{dy}{dx}$ **[1]**

a) x log x b) 0

c) $\frac{y}{x} \cdot \left(\frac{x \log y - y}{y \log x - x}\right)$ d) 1

29. If $f(x) = x^2 + \frac{x^2}{1+x^2} + \frac{x^2}{(1+x^2)^2} + \ldots + \frac{x^2}{(1+x^2)^n} + \ldots$ then at x = 0, f(x)ṁ **[1]**

a) has no limit b) in continuous but not differentiable

c) is discontinuous d) is differentiable

30. Let f(x) = |sin x|; $0 \leq x \leq 2\pi$ then **[1]**

a) f(x) is discontinuous at 3 points b) f(x) is differentiable function at infinite number of points

c) f(x) is non-differentiable at 3 points and continuous everywhere d) f(x) is discontinuous everywhere

31. The differential coefficient of f(log x) with respect to x, where f(x) = log x is **[1]**

a) $(x \log x)^{-1}$ b) $\frac{2x}{\log x}$

c) $\frac{x}{\log x}$ d) $\frac{\log x}{x}$

32. If $y = \sin^{-1}\left\{\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right\}$ then $\frac{dy}{dx} = ?$ **[1]**

a) $\frac{1}{2\sqrt{1-x^2}}$ b) $\frac{-1}{2(1-x^2)}$

c) $\frac{1}{2(1+x^2)}$ d) $\frac{-1}{2\sqrt{1-x^2}}$

33. At x = 2, f(x) = [x] is **[1]**

a) Continuous but not differentiable b) None of these

c) Continuous as well as differentiable d) Differentiable but not continuous

34. The derivative of sin x w.r.t. cos x is **[1]**

a) -tan x b) cot x

c) -cot x d) tan x

35. The differentiation of $\cos^{-1}(5x^2 + 4)$ w.r.t. x is **[1]**

a) $\sqrt{1 + (5x^2 - 4)^2}$ b) $\sqrt{1 - (5x^2 + 4)^2}$

c) $-10x\sqrt{1 - (5x^2 + 4)^2}$ d) $\frac{-10x}{\sqrt{1-(5x^2+4)^2}}$

36. If $y = \sin^{-1}x$, then $(1 - x^2)y_2$ is equal to **[1]**

a) xy_2 b) xy_1

c) xy d) x^2

37. Differentiation of the following w.r.t. $xy = e^{x^3}$ **[1]**

a) $3x^2e^{x^3}$ b) $x^2e^{x^3}$

c) $x^3e^{x^3}$ d) $x^2e^{x^2}$

38. The value of k for which function f(x) = $\begin{cases} x^2, & x \geq 0 \\ kx, & x < 0 \end{cases}$ is differentiable at x = 0 is: **[1]**

a) 1 b) 0

c) 2

d) any real number

39. If $y^{1/n} + y^{-1/n} = 2x$, then $(x^2 - 1)\, y_2 + xy_1 =$ **[1]**

a) 0

b) 1

c) n^2y

d) $-n^2y$

40. If $y^2 = ax^2 + b$, then $\frac{d^2y}{dx^2}$ is equal to **[1]**

a) $\frac{ab}{x^2}$

b) $\frac{ab}{y^2}$

c) $\frac{ab}{y^3}$

d) $\frac{ab}{x^3}$

41. If y + sin y = cos x, then $\frac{dy}{dx}$ is equal to **[1]**

a) $-\frac{\sin x}{1+\cos y}, y = (2n+1)\pi$

b) $-\frac{\sin x}{1+\cos y}, y \neq (2n+1)\pi$

c) $\frac{\sin x}{1+\cos y}, y \neq (2n+1)\pi$

d) $-\frac{\sin x}{1-\cos y}, y \neq (2n-1)\pi$

42. Let f(x) = $\begin{cases} \left|\tan\left(\frac{\pi}{4}+x\right)\right|^{\frac{1}{x}}, & x \neq 0 \\ k, & x = 0 \end{cases}$ then the value of k such that f(x) holds continuity at x = 0 is **[1]**

a) e^2

b) $\frac{1}{e^2}$

c) e

d) e^{-2}

43. The function f(x) = $\begin{cases} 1+x, \text{ when } x \leq 2 \\ 5-x, \text{ when } x > 2 \end{cases}$ is continuous and differentiable at x=2 ,yes or no **[1]**

a) Differentiable but not continuous at x = 2

b) Continuous as well as differentiable at x = 2

c) Continuous but not differentiable at x = 2

d) Differentiable but continuous at x = 2

44. If y = $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ then $\frac{dy}{dx} =$? **[1]**

a) $\frac{-2}{(1-x^2)}$

b) $\frac{-2}{\sqrt{1-x^2}}$

c) $\frac{-2}{(1+x^2)}$

d) $\frac{2}{\sqrt{1-x^2}}$

45. If y = $f\left(\frac{3x+4}{5x+6}\right)$ and f'(x) = $\tan x^2$ then $\frac{dy}{dx}$ is equal to **[1]**

a) -2 $\tan\left(\frac{3x+4}{5x+3}\right) \times \frac{1}{(5x+3)^2}$

b) $\tan x^2$

c) $f\left(\frac{3\tan x^2+4}{5\tan x^2+6}\right)$

d) -2 $\tan\left(\frac{3x+4}{5x+6}\right) \times \frac{1}{(5x+6)^2}$

46. Number of points at which f(x) = $\frac{1}{\log|x|}$ is discontinuous is **[1]**

a) 1

b) 3

c) 2

d) 4

47. The function f(x) = $\begin{cases} \frac{\sin x}{x} + \cos x, \text{ if } x \neq 0 \\ k \quad , \text{ if } x = 0 \end{cases}$ is continuous at x = 0, then the value of k is **[1]**

a) 1

b) 3

c) 1.5

d) 2

[1]

48. The value of p and q for which the function f(x) = $\begin{cases} \frac{\sin(p+1)x+\sin x}{x} & ,x<0 \\ q & ,x=0 \\ \frac{\sqrt{x+bx^2}-\sqrt{x}}{x^{\frac{3}{2}}} & ,x>0 \end{cases}$ is continuous for all x ∈ R, are

a) p = $-\frac{3}{2}$, q = $\frac{1}{2}$

b) p = $-\frac{3}{2}$, q = $-\frac{1}{2}$

c) p = $\frac{5}{2}$, q = $\frac{7}{2}$

d) p = $\frac{1}{2}$, q = $\frac{3}{2}$

49. If $\sqrt{x}+\sqrt{y}=\sqrt{a}$ then $\frac{dy}{dx}$ = ? **[1]**

a) $\frac{-\sqrt{x}}{\sqrt{y}}$

b) $\frac{-\sqrt{y}}{\sqrt{x}}$

c) $\frac{1}{2}\frac{-\sqrt{y}}{\sqrt{x}}$

d) $\frac{1}{2}\frac{\sqrt{y}}{\sqrt{x}}$

50. If $\sqrt{x}+\sqrt{y}=\sqrt{a}$, then $\left(\frac{d^2y}{dx^2}\right)_{x=a}$ is equal to **[1]**

a) $\frac{1}{2a}$

b) a

c) $\frac{1}{4a}$

d) $\frac{1}{a}$

51. The value of k for which f(x) $\begin{cases} \frac{3x+4\tan x}{x}, & \text{when } x \neq 0 \\ k, & \text{when } x=0 \end{cases}$ is continuous at x = 0, is **[1]**

a) 3

b) 7

c) 8

d) 4

52. If $y=x\sqrt{1-x^2}+\sin^{-1}x$, then $\frac{dy}{dx}$ is equal to **[1]**

a) $\frac{1}{\sqrt{1-x^2}}$

b) $\sqrt{1-x^2}$

c) $2\sqrt{1-x^2}$

d) $4\sqrt{1-x^2}$

53. The differential coefficient of sin (cos (x^2)) w.r.t. x is **[1]**

a) -2xsin(x^2) cos(cos x^2)

b) 2xsin(x^2) cos(x^2)

c) 2xcos(x^2) sin(x^2)

d) 2xsin(x^2) cos(x^2) cos x

54. If $x=a(\cos\theta+\theta\sin\theta)$ and $y=a(\sin\theta-\theta\cos\theta)$ $\frac{dy}{dx}$ = ? **[1]**

a) a cot θ

b) cot θ

c) tan θ

d) a tan θ

55. Let f(x) be a polynomial. Then, the second-order derivative of f(e^x) is **[1]**

a) f "(e^x) e^x + f '(e^x)

b) f "(e^x) e^{2x} + f "(e^x) e^x

c) f "(e^x) e^{2x} + f '(e^x) e^x

d) f "(e^x)

56. Let f(x) = x - |x| then f(x) is **[1]**

a) continuous ∀ x ∈ R and not differentiable at x = 0

b) neither continuous nor differentiable at x = 0

c) discontinuous at x = 0

d) differentiable ∀ x ∈ R

57. The function f(x) = $\sin^{-1}$(cos x) is **[1]**

a) Not differentiable at x = 0

b) differentiable at x = 0

c) discontinuous at x = 0

d) continuous at x = 0

58. The derivative of the function $\cot^{-1}\left\{(\cos 2x)^{1/2}\right\}$ at x = π/6 is **[1]**

a) $(2/3)^{1/2}$

b) $(1/3)^{1/2}$

c) $3^{1/2}$

d) $6^{1/2}$

59. The value of k for which f(x) = $\begin{cases} 3x+5, & x \geq 2 \\ kx^2, & x < 2 \end{cases}$ is a continuous function, is: **[1]**

a) $\frac{11}{4}$

b) $\frac{4}{11}$

c) $-\frac{11}{4}$

d) 11

60. If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ is **[1]**

a) $\frac{1-\log x}{1+\log x}$

b) $\frac{1+x}{1+\log x}$

c) $\frac{\log x}{(1+\log x)^2}$

d) not defined

61. If $y = \left(\frac{x}{n}\right)^{nx}$ $(1 + \log\frac{x}{n})$, y'(n) is given by **[1]**

a) $\frac{n^2+1}{n}$

b) $\left(\frac{1}{n}\right)^n$

c) $\frac{1}{n}$

d) $\left(\frac{1}{n}\right)^n \frac{n^2+1}{n}$

62. If $y = \tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$ then $\frac{dy}{dx} = ?$ **[1]**

a) $\frac{1}{2}$

b) $-\frac{1}{2}$

c) $\frac{-1}{2}$

d) $\frac{1}{(1+x^2)}$

63. If $x = \log(1 + t^2)$ and $y = t - \tan^{-1} t$, then $\frac{dy}{dx}$ is equal to **[1]**

a) $t^2 - 1$

b) $e^x - 1$

c) $e^x - y$

d) $\frac{\sqrt{e^x-1}}{2}$

64. If $f(x) = \frac{\sin 4\pi[\pi^2 x]}{7+[x]^2}$, $[\cdot]$ denotes the greatest integer function, then f(x) is **[1]**

a) (x) exists but f'(x) does not exist for some value of x.

b) continuous for all x but f(x) does not exist

c) discontinuous at some x

d) f'(x) exists for all x

65. If f (x) = $x \tan^{-1} x$ then f ' (1) is equal to **[1]**

a) $\frac{1}{2} - \frac{\pi}{2}$

b) $\frac{1}{2} - \frac{\pi}{4}$

c) $\frac{\pi}{4} - \frac{1}{2}$

d) $\frac{\pi}{4} + \frac{1}{2}$

66. If $y = \frac{\log x}{x}$, then $\frac{d^2y}{dx^2} =$ **[1]**

a) $\frac{2\log x-3}{x^3}$

b) $\frac{2\log x+3}{x^3}$

c) $\frac{2\log x-3}{x^4}$

d) $\frac{3-2\log x}{x^3}$

67. An oblique asymptote to the curve $y = x + e^{-x} \sin x$ is **[1]**

a) y = x + e

b) y = x + $\frac{1}{\pi}$

c) y = x

d) y = x + π

68. If u = $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ and v = $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$, then $\frac{du}{dv}$ is **[1]**

a) $\frac{1}{2}$ b) $\frac{1-x^2}{1+x^2}\{4,-4\},\phi$

c) 1 d) x

69. The function f(x) = x|x| is **[1]**

a) continuous and differentiable at x = 0. b) differentiable but not continuous at x = 0.

c) continuous but not differentiable at x = 0. d) neither differentiable nor continuous at x = 0.

70. If f(x), g(x), h(x) are polynomials in x of degree 2 and F(x) = $\begin{vmatrix} f(x) & g(x) & h(x) \\ f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix}$, then F' (x) is equal to **[1]**

a) 0 b) -2

c) -1 d) 2

71. The function $f(x) = \begin{cases} \frac{\sin 3x}{x}, & x \neq 0 \\ \frac{k}{2}, & x = 0 \end{cases}$ is continuous at x = 0, then k = **[1]**

a) 3 b) 12

c) 9 d) 6

72. If y = $\cot^{-1}\left(\frac{1-x}{1+x}\right)$ then $\frac{dy}{dx}$ = ? **[1]**

a) $\frac{1}{(1+x^2)}$ b) $\frac{-1}{(1+x^2)}$

c) $\frac{1}{(1-x^2)^{3/4}}$ d) $\frac{1}{(1+x^2)^{3/2}}$

73. Let g(x) = $\begin{cases} e^{2x}, & x < 0 \\ e^{-2x}, & x \geq 0 \end{cases}$ then g(x) does not satisfy the condition **[1]**

a) differentiable at x = 0 b) continuous $\forall$ x $\in$ R

c) continuous $\forall$ x $\in$ R and non differentiable at x = ± 1 d) not differentiable at x = 0

74. Let $f(x) = \begin{cases} \frac{1}{|x|} & \text{for } |x| \geq 1 \\ ax^2 + b & \text{for } |x| < 1 \end{cases}$ If f(x) is continuous and differentiable at any point, then **[1]**

a) a = 1, b = –1 b) $a = \frac{1}{2}, b = -\frac{3}{2}$

c) $a = \frac{1}{2}, b = \frac{3}{2}$ d) none of these

75. If y = $\cos^{-1}$ x, then the value of $\frac{d^2y}{dx^2}$ in terms of y alone is **[1]**

a) cosec y cos y b) -cot y cosec y

c) cosec y $\cot^2$ y d) -cot y cosec^2 y