Solution

CET25M5 CONTINUITY AND DIFFERENTIABILITY

Class 12 - Mathematics

1.

(c) f '(a^+) = ϕ (a)

Explanation: Given that, f(x) = |x - a| ϕ (x), where ϕ(x) continuous function.

|x - a| $\Rightarrow$ x - a if x - a > 0

|x - a| $\Rightarrow$ -(x - a) if x - a < 0

By definition of continuity,

$f'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$

Hence, f '(a^+) = ϕ(x)

2.

(d) 1, 3

Explanation: Here, f(x) = $\begin{cases} 3, & \text{if } 0 \leq x \leq 1 \\ 4, & \text{if } 1 < x < 3 \\ 5, & \text{if } 3 \leq x \leq 10 \end{cases}$

For 0 $\leq$ x $\leq$ 1, f(x) = 3; 1 < x < 3 ; f(x) = 4 and 3 $\leq$ x $\leq$ 10, f(x) = 5 are constant functions, so it is continuous in the given interval, so we have to check the continuity at x = 1, 3.

At x = 1, LHL = $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^-} (3)$ = 3,

RHL = $\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^+} (4)$ = 4

$\therefore$ LHL $\neq$ RHL

Thus, f(x) is discontinuous at x = 1.

At x = 3, LHL = $\lim\limits_{x \to 3^-} f(x) = \lim\limits_{x \to 3^-} (4)$ = 4,

RHL = $\lim\limits_{x \to 3^+} f(x) = \lim\limits_{x \to 3^+} (5)$ = 5

$\therefore$ LHL $\neq$ RHL.

Thus, f(x) is discontinuous at x = 3.

Hence, f (x) is continuous everywhere except at x = 1, 3.

3.

(b) has jump discontinuity

Explanation: has jump discontinuity

4. **(a)** differentiable everywhere except at x = 0

Explanation: We have, f(x) = $e^{|x|}$ = $\begin{cases} e^x, & x \geq 0 \\ e^{-x}, & x < 0 \end{cases}$

LHD = $\lim\limits_{h \to 0} \frac{f(0-h)-f(0)}{-h}$

= $\lim\limits_{h \to 0} \frac{e^{-(0-h)}-e^0}{-h}$ = $\lim\limits_{h \to 0} \frac{e^h-1}{-h}$ = -1 and RHD = $\lim\limits_{h \to 0} \frac{f(0+h)-f(0)}{h}$

= $\lim\limits_{h \to 0} \frac{e^{0+h}-f(0)}{h}$ = $\lim\limits_{h \to 0} \frac{e^h-1}{h}$ = 1

$\therefore$ LHD $\neq$ RHD

So, f(x) is not differentiable at x = 0.

Hence, f(x) is differentiable everywhere except at x = 0.

5.

(d) 0

Explanation: y = $\tan^{-1}$ x + $\cot^{-1}$ x + $\sec^{-1}$ x + cosec^{-1} x

= $\frac{\pi}{2} + \frac{\pi}{2}$ $\left[\because \tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}\right]$

$\left[\sec^{-1} x + \text{cosec}^{-1} x = \frac{\pi}{2}\right]$

$\therefore$ y = $\pi \Rightarrow \frac{dy}{dx}$ = 0

6.

(c) f(x) and g(x) both are continuous at x = 0

Explanation: Given f(x) = |x| and g(x) = $|x^3|$,

$f(x)=\begin{cases}-x, & x\leq 0\\ x, & x>0\end{cases}$

Checking differentiability and continuity,

LHL at x =0,

$\lim\limits_{x\to 0^-} f(x)=\lim\limits_{h\to 0} f(0-h)=\lim\limits_{h\to 0}-(0-h)=0$

RHL at x =0,

$\lim\limits_{x\to 0^+} f(x)=\lim\limits_{h\to 0} f(0+h)=\lim\limits_{h\to 0}(0+h)=0$

And f(0)=0

Hence, f(x) is continuous at x =0.

LHD at x =0,

$\lim\limits_{x\to 0^-}\frac{f(x)-f(0)}{x-0}=\lim\limits_{h\to 0}\frac{f(0-h)-f(0)}{0-h-0}$

$=\lim\limits_{h\to 0}\frac{(0-h)-(0)}{-h}=-1$

RHD at x =0,

$\lim\limits_{x\to 0^+}\frac{f(x)-f(0)}{x-0}=\lim\limits_{h\to 0}\frac{f(0+h)-f(0)}{0+h-0}$

$=\lim_{h\to 0}\frac{(0+h)-(0)}{h}=1$

$\because$ LHD $\neq$ RHD

$\therefore$ f(x) is not differentiable at x =0.

$g(x)=\begin{cases}-x^3, & x\leq 0\\ x^3, & x>0\end{cases}$

Checking differentiability and continuity,

LHL at x =0,

$\lim\limits_{x\to 0^-} g(x)=\lim\limits_{h\to 0} g(0-h)=\lim\limits_{h\to 0}-(0-h)^3=0$

RHL at x =0,

$\lim\limits_{x\to 0^+} g(x)=\lim\limits_{h\to 0} g(0+h)=\lim\limits_{h\to 0}(0+h)^3=0$

And g(0)=0

Hence, g(x) is continuous at x =0.

LHD at x =0,

$\lim\limits_{x\to 0^-}\frac{g(x)-g(0)}{x-0}=\lim\limits_{h\to 0}\frac{g(0-h)-g(0)}{0-h-0}$

$=\lim\limits_{h\to 0}\frac{(0-h)^3-(0)}{-h}=0$

RHD at x =0,

$\lim\limits_{x\to 0^+}\frac{g(x)-g(0)}{x-0}=\lim\limits_{h\to 0}\frac{g(0+h)-g(0)}{0+h-0}$

$=\lim\limits_{h\to 0}\frac{(0+h)^3-(0)}{h}=0$

$\because$ LHD = RHD

$\therefore$ g(x) is differentiable at x =0.

7.

(b) $-a^{1/2}$

Explanation: $\lim\limits_{x\to 0}\frac{\sqrt{a^2-ax+x^2}-\sqrt{a^2+ax+x^2}}{\sqrt{a+x}-\sqrt{a-x}}$

$=\lim\limits_{x\to 0}\frac{\sqrt{a^2-ax+x^2}-\sqrt{a^2+ax+x^2}}{\sqrt{a+x}-\sqrt{a-x}}\times\frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}+\sqrt{a-x}}$

$=\lim\limits_{x\to 0}\frac{\sqrt{a^2-ax+x^2}-\sqrt{a^2+ax+x^2}}{a+x-(a-x)}$

$x\frac{(\sqrt{a+x}+\sqrt{a-x})\sqrt{a^2-ax+x^2}+\sqrt{a^2+ax+x^2}}{\sqrt{a^2-ax+x^2}+\sqrt{a^2+ax+x^2}}$

$=\lim\limits_{x\to 0}\frac{a^2-ax+x^2-(a^2+ax+x^2)}{2x}\times\frac{\sqrt{a+x}+\sqrt{a}}{\sqrt{a^2-ax+x^2}+\sqrt{a^2}}$

$= \lim_{x \to 0} \frac{-2ax}{2x} \times \frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a^2-ax+x^2}+\sqrt{a^2+ax+x^2}}$
$= -\frac{2a\sqrt{a}}{a+a}$
$= -\frac{2a\sqrt{a}}{2a}$
$= -\sqrt{a}$

8. **(a)** -sec$^2\left(\frac{\pi}{4} - \text{x}\right)$

Explanation: -sec$^2\left(\frac{\pi}{4} - \text{x}\right)$

y = $\frac{\cos \text{x} - \sin \text{x}}{\cos \text{x} + \sin \text{x}}$

y = $\frac{cosx(1-\frac{sinx}{cosx})}{cosx(1+\frac{sinx}{cosx})}$

y = $\frac{1-tanx}{1+tanx}$

y = $\frac{tan\frac{\pi}{4}-tanx}{1+tan\frac{\pi}{4}tanx}$

y = tan ($\frac{\pi}{4}$-x)

$\frac{dy}{dx} = -sec^2(\frac{\pi}{4} - x)$

9. **(a)** $\frac{2}{1+x^2}$

Explanation: Given, y = sin$^{-1}\left(\frac{2x}{1+x^2}\right)$

On differentiating both sides w.r.t. x, we get

$\frac{dy}{dx}$ = $\frac{1}{\sqrt{1-\left(\frac{2x}{1+x^2}\right)^2}} \frac{d}{dx}\left(\frac{2x}{1+x^2}\right)$

= $\frac{1+x^2}{\sqrt{(1+x^2)^2-4x^2}} \left[\frac{(1+x^2)(2)-2x(2x)}{(1+x^2)^2}\right]$

= $\frac{1+x^2}{\sqrt{1+x^4+2x^2-4x^2}} \left[\frac{2-2x^2}{(1+x^2)^2}\right]$

= $\frac{2(1+x^2)(1-x^2)}{(1-x^2)(1+x^2)^2}$ = $\frac{2}{1+x^2}$

10.

(d) |sec θ|

Explanation: $x = a\cos^3\theta \Rightarrow \cos^2\theta = \left(\frac{x}{a}\right)^{\frac{2}{3}}$

$y = a\sin^3\theta \Rightarrow \sin^2\theta = \left(\frac{y}{a}\right)^{\frac{2}{3}}$

We know that

cos$^2\theta$ + sin$^2\theta$ = 1

$\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{a}\right)^{\frac{2}{3}} = 1$

$x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{x^3}$

Differentiating with respect to x,

$\frac{2}{3}x^{\frac{-1}{3}} + \frac{2}{3}y^{\frac{-1}{3}}\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \left(\frac{y}{x}\right)^{\frac{1}{3}}$

$\sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{1+\left[\left(\frac{y}{x}\right)^{\frac{1}{3}}\right]^2}$

$\sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{1+\left[\left(\frac{\sin^3\theta}{\cos^3\theta}\right)^{\frac{1}{3}}\right]^2}$

$\sqrt{1+\left(\frac{dy}{dx}\right)^2} = |\sec\theta|$.

Which is the required solution.

11.

(c) x$^{\text{x}}$\{(1 + log x)^2 + $\frac{1}{x}$ \}

Explanation: y = $x^x = e^{x \log x}$

$\therefore \frac{dy}{dx} = e^{x \log x}(x \times \frac{1}{x} + \log x)$

$= e^{x \log x}(1 + \log x)$

$\therefore \frac{d^2y}{dx^2} = e^{x \log x}\left(\frac{1}{x}\right) + (1 + \log x)\, e^{x \log x}(1 + \log x)$

$= \frac{x^x}{x} + (1 + \log x)^2 e^x \log x$

$= x^{x-1} + (1 + \log x)^2 x^x = x^x \{(1 + \log x)^2 + \frac{1}{x}\}$

12. **(a)** $x^{\sin x}\left\{\frac{\sin x + x \log x \cdot \sin x}{x}\right\}$

Explanation: Let y = f(x) = $x^{\sin x}$

Taking log both sides, we obtain

$\log_e y = \sin x \log_e x$ -(1) (Since $\log_a b^c = c \log_a b$)

Differentiating (i) with respect to x, we obtain

$\frac{1}{y}\frac{dy}{dx} = \sin x \times \frac{1}{x} + \log_e x \times \cos x$

$\Rightarrow \frac{dy}{dx} = y \times \left(\frac{\sin x}{x} + \log_e x \cos x\right)$

$\Rightarrow \frac{dy}{dx} = f'(x) = x^{sinx}\left(\frac{\sin x + x \log x \sin x}{x}\right).$

Which is the required solution.

13.

(b) has removable discontinuity

Explanation: has removable discontinuity

14. **(a)** – tan t

Explanation: We have to find: $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3a\sin^2 t \cos t}{3a\cos^2 t(-\sin t)} = -\tan t$

15. **(a)** 2^n

Explanation: $f(x) = x^n$

$f(1) + \frac{f^1(1)}{1!} + \frac{f^2(1)}{2!} + \frac{f^3(1)}{3!} + \ldots + \frac{f^n(1)}{n!}$

$= 1 + \frac{n}{1!} + \frac{n(n-1)}{2!} + \frac{n(n-1)(n-2)}{3!} + \ldots + \frac{n(n-1)\ldots 1}{n!}$

$= (1 + 1)^n = 2^n$ [By using Binomial expansion]

16.

(b) $\frac{y}{x}$

Explanation: We have, $\sin^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right) = \log a$

$\Rightarrow \frac{x^2-y^2}{x^2+y^2} = \sin \log a$

$\Rightarrow \frac{(x^2+y^2)\left(2x-2y\frac{dy}{dx}\right)-(x^2-y^2)\left(2x+2y\frac{dy}{dx}\right)}{(x^2+y^2)^2} = 0$

$\Rightarrow \frac{2x^3-2x^2y\frac{dy}{dx}+2xy^2-2y^3\frac{dy}{dx}-2x^3-2x^2y\frac{dy}{dx}+2xy^2+2y^3\frac{dy}{dx}}{(x^2+y^2)^2} = 0$

$\Rightarrow -4x^2y\frac{dy}{dx} + 4xy^2 = 0$

$\Rightarrow -4x^2y\frac{dy}{dx} = -4xy^2$

$\Rightarrow \frac{dy}{dx} = \frac{4xy^2}{4x^2y}$

$\therefore \frac{dy}{dx} = \frac{y}{x}$.

Which is the required solution.

17.

(d) {0, 1}

Explanation: Given: $f(x) = \frac{1}{1-x}$

Clearly,

$f: R-\{1\} \rightarrow R$

Now, $f(f(x))=f\left(\frac{1}{1-x}\right)=\left(\frac{1}{1-\left(\frac{1}{1-x}\right)}\right)=\left(\frac{1-x}{-x}\right)=\left(\frac{x-1}{x}\right)$

$\therefore$ fof :

$R-\{0,1\} \rightarrow R$

Now,

$f(f(f(x)))=f\left(\frac{x-1}{x}\right)=\left(\frac{1}{1-\left(\frac{x-1}{x}\right)}\right)=x$

$\therefore$ fofof:

$R-\{0,1\} \rightarrow R$

Thus, $f(f(f(x)))$ is not defined at x = 0, 1

Hence, $f(f(f(x)))$ is discontinuous at {0, 1}

18. **(a)** 8

Explanation: Since, f(x) is continuous at x = 0, then

LHL = RHL = f(0)

or LHL = RHL = k

Now, $\text{LHL}=\lim_{h \rightarrow 0} \frac{e^{3(0-h)}-e^{-5(0-h)}}{0-h}$

$=\lim_{h \rightarrow 0} \frac{e^{-3h}-e^{5h}}{-h}$

$=\lim_{h \rightarrow 0}\left(\frac{e^{-3h}-1}{-h}\right)+\lim_{h \rightarrow 0}\left(\frac{e^{5h}-1}{h}\right)$

$=3 \lim_{h \rightarrow 0}\left(\frac{e^{-3k}-1}{-3h}\right)+5 \lim_{h \rightarrow 0}\left(\frac{e^{5h}-1}{5h}\right)$

= 3 × 1 + 5 × 1 = 8

Thus, k = 8

19.

(b) -10

Explanation: Given, f''(x) = g''(x)

On integrating both sides, we get

f'(x) = g'(x) + c ⇒ f'(1) = f'(1) + r ⇒ 4 = 6 + c ⇒ c = -2

∴ f'(x) = g'(x) - 2

Again, on integrating both sides, we get

f(x) = g(x) - 2x + c_1 ⇒ f(2) = g(2) - 2 × 2 + c_1

⇒ 3 = 9 - 4 + c_1 ⇒ c_1 = - 2

∴ f(x) - g(x) = -2x - 2

At x = 4, [f(x) - g(x)] = - 8 - 2 = -10

20. **(a)** continuous at non-integer points only

Explanation: Given function f(x) = x – [x]

For any integer n,

$$\mathrm{f(x)}=\left\{\begin{array}{c}\mathrm{x-(n-1), n-1 \leq x<n} \\ \mathrm{0, x=n} \\ \mathrm{x-n, n \leq x<n+1}\end{array}\right\}$$

LHL: $\lim_{x \rightarrow n^{-}}$ x - n + 1 = n - n + 1 = 1

RHL: $\lim_{x \rightarrow 1^{+}}$ x - n = n - n = 0

Hence, f(x) is not continuous at integer points.

∴ Given function is continuous on non-integer points only.

21. **(a)** (1 + sin 2x) y_1

Explanation: y = $e^{\tan x}$

y_1 = $\sec^2 x\, e^{\tan x}$

⇒ $\cos^2 x y_1 = e^{\tan x}$

Again differentiating w.r.t. x we get

$\cos^2(x).y_2 - 2\cos x \sin x y_1 = \sec^2 x e^{\tan x}$

$\Rightarrow \cos^2(x).y_2 = y_1 \sin 2x + y_1.$

22. **(a)** $-e^x \tan(e^x)$

Explanation: Let y = log (cos e^x)

On differentiating both sides w.r.t. x, we get

$\frac{dy}{dx} = \frac{d}{dx}[\log\{\cos(e^x)\}]$

$= \frac{1}{\cos(e^x)} \frac{d}{dx}\{\cos(e^x)\}$ [using chain rule]

$= \frac{1}{\cos(e^x)}\{-\sin(e^x)\} \frac{d}{dx}(e^x)$[using chain rule]

$= -\tan(e^x) \cdot e^x = -e^x \tan(e^x)$

23.

(c) a function of y only

Explanation: $y = ax^2 + bx + c$

$\frac{dy}{dx} = 2ax + b$

$\frac{d^2y}{dx^2} = 2a$

$y^3 \frac{d^2y}{dx^2} = 2ay^3$ = A function of y only

24.

(d) 1

Explanation: $\because \lim_{x\to 0}$ f(x) exists $\Rightarrow \lim_{x\to 0^-}$ f(x) = $\lim_{x\to 0^+}$ f(x)

$\Rightarrow \lim_{h\to 0}$ f(0 - h) = $\lim_{h\to 0}$ f(0 + h)

$\Rightarrow \lim_{h\to 0}$ f(-h) = $\lim_{h\to 0}$ f(h)

$\Rightarrow \lim_{h\to 0}\{|-h| + a\} = \lim_{h\to 0}$ cos [h]

$\Rightarrow$ a = cos 0 $\Rightarrow$ a = 1

25.

(c) $\frac{2x}{(1+x^4)}$

Explanation: Given that $y = \tan^{-1}\left(\frac{1+x^2}{1-x^2}\right)$

Let $x^2 = \tan\theta$

$\Rightarrow \theta = \tan^{-1} x^2$

Hence, $y = \tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right)$

Using $\tan\left(\frac{\pi}{4} + x\right) = \frac{1+\tan x}{1-\tan x}$, we obtain

$y = \tan^{-1}\tan\left(\frac{\pi}{4} + \theta\right) = \frac{\pi}{4} + \theta = \frac{\pi}{4} + \tan^{-1}(x^2)$

Differentiating with respect to x, we obtain

$\frac{dy}{dx} = \frac{1}{1+x^4} \times 2x = \frac{2x}{1+x^4}$

26.

(b) π

Explanation: π

27.

(b) $n = \frac{m\pi}{2}$

Explanation: We have, $f(x) = \begin{cases} mx + 1, & \text{if } x \leq \frac{\pi}{2} \\ \sin x + n, & \text{if } x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$

$\therefore LHL = \lim_{x\to\frac{\pi}{2}} (mx + 1) = \lim_{h\to 0}\left[m\left(\frac{\pi}{2} - h\right) + 1\right] = \frac{m\pi}{2} + 1$

and $RHL = \lim_{x\to\frac{\pi'}{2}} (\sin x + n) = \lim_{h\to\infty}\left[\sin\left(\frac{\pi}{2} + h\right) + n\right]$

$= \lim_{n\to 0} \cos h + n = 1 + n$

Since the function is continuous, we have

$LHL = RHL$

$\Rightarrow m \cdot \frac{\pi}{2} + 1 = n + 1$

$\therefore\ n = m \cdot \frac{\pi}{2}$

28.

(c) $\frac{y}{x} \cdot \left(\frac{x \log y - y}{y \log x - x}\right)$

Explanation: $x^y = y^x \Rightarrow$ y log x = x log y

$\frac{y}{x}$ + log x $\frac{dy}{dx} = \frac{x}{y}\frac{dy}{dx}$ + log y

$\Rightarrow \left(\log x - \frac{x}{y}\right)\frac{dy}{dx} = \left(\log y - \frac{y}{x}\right) = \frac{x \log y - y}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{x \log y - y}{y \log x - x} \times \frac{y}{x} = \frac{y}{x} \times \frac{x \log y - y}{y \log x - x}$

29.

(c) is discontinuous

Explanation: Given $f(x) = x^2 + \frac{x^2}{1+x^2} + \frac{x^2}{(1+x^2)^2} + \ldots\ldots + \frac{x^2}{(1+x^2)^n} + \cdots$

For x =0, $x^2 = 0$

$\Rightarrow$ f(x) = 0

For x $\neq$ 0,

$x^2 + 1 > x^2$

$\Rightarrow 0 < \frac{x^2}{1+x^2} < 1$

$\therefore \lim_{x\to 0} f(x) = \lim_{x\to 0}\ x^2 + \frac{x^2}{1+x^2} + \frac{x^2}{(1+x^2)^2} + \cdots\ldots + \frac{x^2}{(1+x^2)^n} + \cdots$

$\therefore \lim_{x\to 0} f(x) = \lim_{x\to 0} x^2 \left(1 + \frac{1}{1+x^2} + \frac{1}{(1+x^2)^2} + \cdots\ldots + \frac{1}{(1+x^2)^n} + \cdots\right)$

$\therefore \lim_{x\to 0} f(x) = \lim_{x\to 0} \left\{x^2\left(\frac{1}{1-\frac{1}{1+x^2}}\right)\right\}$

$\because$ Sum of infinite series where $r = \frac{1}{1+x^2}$

$\Rightarrow \lim_{x\to 0} f(x) = \lim_{x\to 0} x^2\left(\frac{1+x^2}{x^2}\right)$

$\Rightarrow \lim_{x\to 0} f(x) = \lim_{x\to 0} x^2 + 1$

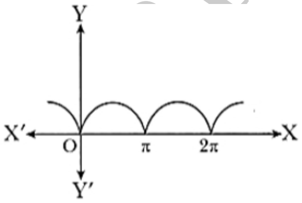
$\Rightarrow \lim_{x\to 0} f(x) = 1 \neq f(0)$

So, f(x) is discontinuous at x = 0

30.

(c) f(x) is non-differentiable at 3 points and continuous everywhere

Explanation:



It is clear from graph that f(x) is continuous everywhere in $0 \leq x \leq 2\pi$. And has sharp edge at x = 0, π, 2π so it is not differentiable at x = 0, π, 2π.

Because it has no unique tangents.

31. **(a)** $(x \log x)^{-1}$

Explanation: We have, f(x) = log x

$\Rightarrow$ f(log x) = log(log x)

$\Rightarrow f'(\log x) = \frac{1}{\log x}\frac{d}{dx}(\log x)$

$\Rightarrow f'(\log x) = \frac{1}{x \log x}$

$\Rightarrow$ f'(log x) = $(x \log x)^{-1}$

32.

(d) $\frac{-1}{2\sqrt{1-x^2}}$

Explanation: Put x = cos 2θ ,we get

$\Rightarrow y=\sin^{-1}\left(\frac{\sqrt{1+\cos 2\theta}}{2}+\frac{\sqrt{1-\cos 2\theta}}{2}\right)$

$\Rightarrow y=\sin^{-1}\left(\frac{\sqrt{2\cos^2 2\theta}}{2}+\frac{\sqrt{2\cdot\sin^2\theta}}{2}\right)$

$\Rightarrow y=\sin^{-1}\left(\frac{\cos 2\theta}{\sqrt{2}}+\frac{\sin 2\theta}{\sqrt{2}}\right)$

$\Rightarrow y=\sin^{-1}\left(\sin\left(\frac{\pi}{4}+2\theta\right)\right)$

$\Rightarrow y=\frac{\pi}{4}+2\theta$

$\Rightarrow \frac{dy}{d\theta}=2$

Put $\theta=\frac{\cos^{-1}x}{2}$,we get

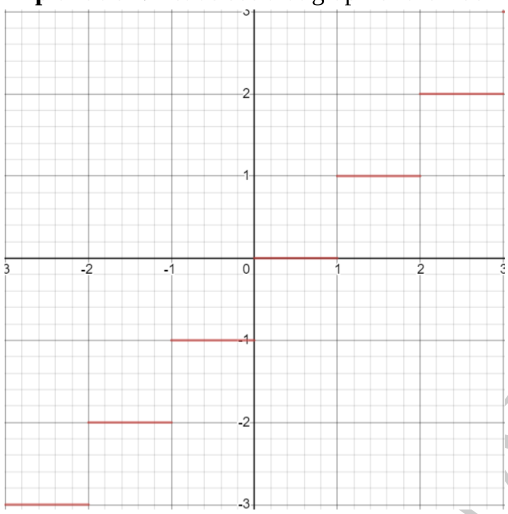
$\Rightarrow \frac{\mathrm{d}\theta}{\mathrm{dx}}=\frac{-1}{4\sqrt{1-\mathrm{x}^2}}$

$\therefore \frac{dy}{dx}=\frac{-1}{2\sqrt{1-x^2}}$

33.

(b) None of these

Explanation: Let us see that graph of the floor function, we get



We can see that f(x) = [x] is neither continuous and non differentiable at x = 2.

34. **(a)** -tan x

Explanation: Let y = sin x and z = cos x

On differentiating y and z w.r.t. x, we get

$\frac{dy}{dx}$ = cos x and $\frac{dz}{dx}$ = -sin x

Now, $\frac{dy}{dz}$ = $\frac{\frac{dy}{dx}}{\frac{dz}{dx}}$ = $\frac{\cos x}{-\sin x}$ = -tan x

35.

(d) $\frac{-10x}{\sqrt{1-\left(5x^2+4\right)^2}}$

Explanation: Let y = $\cos^{-1}(5x^2 + 4)$.

Put z = $5x^2 + 4$.

Then, y = $\cos^{-1}$ z.

On differentiating both sides w.r.t. x, we get

$\frac{dy}{dx}$ = $\frac{d}{dx}\left(\cos^{-1}z\right)$ = $\frac{d}{dz}\left(\cos^{-1}z\right)$ $\frac{dz}{dx}$ [by chain rule]

= $\frac{-1}{\sqrt{1-z^2}}\frac{d}{dx}\left(5x^2+4\right)$ [put z = $5x^2$ + 4]

$= -\frac{1}{\sqrt{1-\left(5x^2+4\right)^2}} \times [5 \times 2x + 0]$ [put z = $5x^2 + 4$]

$= -\frac{10x}{\sqrt{1-\left(5x^2+4\right)^2}}$

36.

(b) xy_1

Explanation: $y = \sin^{-1}x$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} \Rightarrow \sqrt{1-x^2} \cdot \frac{dy}{dx} = 1$

Again, differentiating both sides w.r.to x, we get

$\sqrt{1-x^2} \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \left(\frac{-2x}{2\sqrt{1-x^2}}\right) = 0$

Simplifying, we get $(1 - x^2)y_2 = xy_1$

37. **(a)** $3x^2e^{x^3}$

Explanation: Let y = e^{x^3}

On differentiating both sides w.r.t. x, we get

$\frac{dy}{dx} = e^{x^3}\frac{d}{dx}\left(x^3\right)$[by chain rule of derivative]

$= e^{x^3} \cdot 3x^2 = 3x^2 e^{x^3}$

38.

(b) 0

Explanation: 0

$f(x) = \begin{cases} x^2, & x \geq 0 \\ kx, & x < 0 \end{cases}$

Given function is differentiable

LHD=RHD

$\lim\limits_{h \to 0} \frac{f(0-h)-f(0)}{-h}$ = \lim\limits_ {h\to 0}}\frac{f(0+h)-f(0)}{h}

$\lim\limits_{h \to 0} \frac{k(0-h)-k(0)}{-h} = \lim\limits_{h \to 0} \frac{(0+h)^2-(0)^2}{h}$

$\lim\limits_{h \to 0} \frac{k(-h)}{-h} = \lim\limits_{h \to 0} \frac{(h)^2-(0)^2}{h}$

k = 0

39.

(c) n^2y

Explanation: $y^{1/n} + y^{-1/n} = 2x$

Differentiating both sides we get

$\frac{y_1}{n}\left(y^{\frac{1}{n}-1} - y^{\frac{-1}{n}-1}\right) = 2$

$\Rightarrow y_1\left(y^{\frac{1}{n}} - y^{\frac{-1}{n}}\right) = 2ny$

Again differentiating both sides we get

$y_2\left(y^{\frac{1}{n}} - y^{\frac{-1}{n}}\right) + \frac{y_1}{n}\left(y^{\frac{1}{n}^{-1}} + y^{\frac{-1}{n}-1}\right) = 2ny_1$

$\Rightarrow ny_2\left(y^{\frac{1}{n}} - y^{\frac{-1}{n}}\right) + \frac{y_1^2}{y}\left(y^{\frac{1}{n}} + y^{\frac{-1}{n}}\right) = 2n^2y_1$

$\Rightarrow nyy_2\left(y^{\frac{1}{n}} - y^{\frac{-1}{n}}\right) + 2xy_1^2 = 2n^2yy_1$

$\Rightarrow nyy_2\frac{2ny}{y_1} + 2xy_1^2 = 2n^2yy_1$

$\Rightarrow \frac{n^2y^2y_2}{y_1^2} + xy_1 = n^2y$

$\Rightarrow y_2\frac{\left(y^{\frac{1}{n}} - y^{\frac{-1}{n}}\right)^2}{4} + xy_1 = n^2y$

$\Rightarrow y_2\frac{\left(y^{\frac{1}{n}} + y^{\frac{-1}{n}}\right)^2 - 4}{4} + xy_1 = n^2y$

$\Rightarrow y_2 \frac{4x^2-4}{4} + xy_1 = n^2 y$

$\Rightarrow$ $(x^2 - 1)\ y_2 + xy_1 = n^2y$

40.

(c) $\frac{ab}{y^3}$

Explanation: $y^2 = ax^2 + b \Rightarrow 2y\frac{dy}{dx} = 2ax \Rightarrow y\frac{dy}{dx} = ax$

$\Rightarrow \frac{dy}{dx} = \frac{ax}{y} \Rightarrow \frac{d^2y}{dx^2} = \frac{ya - ax\frac{dy}{dx}}{y^2}$

$= \frac{ya - ax\frac{ax}{y}}{y^2} = \frac{a(y^2 - ax^2)}{y^3} = \frac{ab}{y^3}$

41.

(b) $-\frac{\sin x}{1+\cos y}, y \neq (2n+1)\pi$

Explanation: We have, y + sin y = cos x On differentiating both sides w.r.t. x, we get

$\frac{dy}{dx} + \frac{d}{dx}(\sin y)$ = $\frac{d}{dx}(\cos x)$

$\frac{dy}{dx} + \cos y \cdot \frac{dy}{dx}$ = -sin x $\Rightarrow$ $\frac{dy}{dx} = -\frac{\sin x}{1+\cos y}$

where, y $\neq$ (2n + 1)π

42. **(a)** e^2

Explanation: $\lim_{x\to 0} f(x) = \lim_{x\to 0}\left|\tan\left(\frac{\pi}{4} + x\right)\right|^{\frac{1}{x}}$ = $\lim_{x\to 0}\left|\frac{1+\tan x}{1-\tan x}\right|^{\frac{1}{x}}$

= $\lim_{x\to 0}\left[(1+\tan x)^{\frac{1}{\tan x}} \sqrt[\tan x]{x} \times \lim_{x\to 0}\left[(1-\tan x)^{-\frac{1}{\tan x}}\right]^{\frac{\tan x}{x}}\right.$

= e $\times$ e = e^2 $\left|\because \lim_{x\to 0}[1+x]^{\frac{1}{x}} = e\right|$

$\because$ f(x) is continuous at x = 0

$\therefore \lim_{x\to 0}$ f(x) = f(0)

$\Rightarrow$ e^2 = k $\Rightarrow$ k = e^2

43.

(c) Continuous but not differentiable at x = 2

Explanation: For continuity left hand limit must be equal to right hand limit and value at the point.

Continuity at x = 2.

For continuity at x = 2.

L.H.L = $\lim_{x\to 2^-}(1+x)$ = 3

R.H.L = $\lim_{x\to 2^+}(5-x)$ = 3

f(2) = 1 + 2 = 3

$\therefore$ Therefore,f(x) is continuous at x = 2

Now for differentiability.

$\Rightarrow$ $f'(2^-)$ = $\lim_{x\to 2^-} \frac{f(x)-f(2)}{x-2}$

$\Rightarrow$ $f'(2^-)$ = $\lim_{h\to 0} \frac{f(2-h)-f(2)}{2-h-2}$

$\Rightarrow$ $f'(2^-)$ = $\lim_{h\to 0} \frac{1+2-h-3}{2-h-2} = \lim_{h\to 0} \frac{-h}{-h} = 1$.

$\Rightarrow$ $f'(2^+)$ = $\lim_{x\to 2^-} \frac{f(x)-f(2)}{x-2}$

$\Rightarrow$ $f'(2^+)$ = $\lim_{h\to 0} \frac{f(2+h)-f(2)}{2+h-2}$

$\Rightarrow$ $f'(2^-)$ = $\lim_{h\to 0} \frac{5-(2-h)-3}{2+h-2}$

$= \lim_{h\to 0} \frac{h}{-h}$

= -1

As, $f'(2^-)$ is not equal to $f(2^+)$

$\therefore$ f(x) is continuous but not differentiable at x = 2.

44.

(b) $\frac{-2}{\sqrt{1-x^2}}$

Explanation: $\Rightarrow y=\sec^{-1}\left(\frac{1}{2x^2-1}\right)$

$\Rightarrow \sec y=\frac{1}{2x^2-1}$

$\Rightarrow$ cos y = $2x^2$ - 1

$\Rightarrow$ y = $\cos^{-1}(2x^2 - 1)$

Put x = cos θ ,we get

$\Rightarrow$ y = $\cos^{-1}(2\cos^2\theta - 1)$

$\Rightarrow$ y = $\cos^{-1}(\cos 2\theta)$

$\Rightarrow$ y = 2θ

But $\theta = \cos^{-1}$ x, we get

$\Rightarrow \frac{dy}{dx}=\frac{d\left(\cos^{-1}x\right)}{dx}$

$\Rightarrow \frac{dy}{dx}=2\cdot\frac{d\left(\cos^{-1}x\right)}{dx}$

$\Rightarrow \frac{dy}{dx}=2\cdot\left(\frac{-1}{\sqrt{1-x^2}}\right)$

$\Rightarrow \frac{dy}{dx}=\frac{-2}{\sqrt{1-x^2}}$

45.

(d) -2 tan $\left(\frac{3x+4}{5x+6}\right)\times\frac{1}{(5x+6)^2}$

Explanation: -2 tan $\left(\frac{3x+4}{5x+6}\right)\times\frac{1}{(5x+6)^2}$

46.

(b) 3

Explanation: f(x) = $\frac{1}{\log|x|}$

f(x) is not defined for x = 0, -1 ,1

$\therefore$ f(x) is not continuous at x = 0, -1, 1

47.

(d) 2

Explanation: Since the given function is continuous,

$\therefore$ k = $\lim\limits_{x\to 0}\frac{Sinx}{x}+Cosx$

$\Rightarrow$ k = 1 + 1 = 2

48. **(a)** p = $-\frac{3}{2}$, q = $\frac{1}{2}$

Explanation: p = $-\frac{3}{2}$, q = $\frac{1}{2}$

49.

(b) $\frac{-\sqrt{y}}{\sqrt{x}}$

Explanation: Given that $\sqrt{x}+\sqrt{y}=\sqrt{a}$

Differentiating with respect to x, we obtain

$\frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{y}}\frac{dy}{dx}=0$

Or $\frac{dy}{dx}=-\sqrt{\frac{y}{x}}$

50. **(a)** $\frac{1}{2a}$

Explanation: $\sqrt{x}+\sqrt{y}=\sqrt{a}\ldots\ldots\ldots(1)$

$\Rightarrow \frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{y}}\frac{dy}{dx}=0$

$\Rightarrow \frac{dy}{dx}=-\frac{\sqrt{y}}{\sqrt{x}}\ldots\ldots\ldots(2)$

$\Rightarrow \frac{d^2y}{dx^2}=-\frac{\sqrt{x}\frac{1}{2}y^{-\frac{1}{2}}\frac{dy}{dx}-\sqrt{y}\frac{1}{2}x^{-\frac{1}{2}}}{x}$

$=\frac{-\left(\frac{\sqrt{x}}{2\sqrt{y}}\left(-\frac{\sqrt{y}}{\sqrt{x}}\right)-\frac{\sqrt{y}}{2\sqrt{x}}\right)}{x}$

$=\frac{\sqrt{x}+\sqrt{y}}{2x\sqrt{x}}=\frac{\sqrt{a}}{2x\sqrt{x}.}=\frac{\sqrt{a}}{2a\sqrt{a}}=\frac{1}{2a}$

51.

(b) 7

Explanation: $\Rightarrow f(x)=\frac{3x+4\tan x}{x}$ is continuous at x = 0

$\Rightarrow f(x)=\lim\limits_{x\to 0}\frac{3x+4\tan x}{x}$

$\Rightarrow f(x)=\lim\limits_{x\to 0}\frac{3x}{x}+\frac{4\tan x}{x}$

$\Rightarrow f(x)=3+4\lim\limits_{x\to 0}\frac{\tan x}{x}$

$\Rightarrow$ f(x) = 3 + 4

$\therefore$ k = 7

52.

(c) $2\sqrt{1-x^2}$

Explanation: $y=x\sqrt{1-x^2}+\sin^{-1}(x)$

$\Rightarrow \frac{dy}{dx}=x\left\{\frac{1}{2\sqrt{1-x^2}}(-2x)\right\}+\sqrt{1-x^2}+\frac{1}{\sqrt{1-x^2}}$

$\Rightarrow \frac{dy}{dx}=\frac{-x^2}{\sqrt{1-x^2}}+\sqrt{1-x^2}+\frac{1}{\sqrt{1-x^2}}$

$\Rightarrow \frac{dy}{dx}=\frac{-x^2+1-x^2+1}{\sqrt{1-x^2}}$

$\Rightarrow \frac{dy}{dx}=\frac{-2x^2+2}{\sqrt{1-x^2}}$

$\Rightarrow \frac{dy}{dx}=2\sqrt{1-x^2}$

53. **(a)** $-2x\sin(x^2)\cos(\cos x^2)$

Explanation: $y = \sin(\cos x^2)$

Therefore, $\frac{dy}{dx}=\frac{d}{dx}\sin\left(\cos x^2\right)$

$= \cos(\cos x^2)\frac{d}{dx}(\cos x^2)$

$= \cos(\cos x^2)(-\sin x^2)\frac{d}{dx}(x^2)$

$= -\sin x^2\cos(\cos x^2)(2x)$

$= -2x\sin x^2\cos(\cos x^2)$

54.

(c) $\tan\theta$

Explanation: $x=a(\cos\theta+\theta\sin\theta)$,we get

$\therefore \frac{dx}{d\theta}=a(-\sin\theta+\sin\theta+\theta\cos\theta)$

$\Rightarrow \frac{d\theta}{dx}=\frac{1}{a\theta\cos\theta}$

$y=a(\sin\theta-\theta\cos\theta)$,we get

$\therefore \frac{dy}{d\theta}=a(\cos\theta-(\cos\theta+\theta(-\sin\theta))$

$\Rightarrow \frac{dy}{d\theta}=a\cos\theta-a\cos\theta+\theta a\sin\theta$

$\Rightarrow \frac{dy}{d\theta}=a\theta\sin\theta$

$\Rightarrow \frac{dy}{dx}=\frac{dy}{d\theta}\times\frac{d\theta}{dx}$

$\Rightarrow \frac{dy}{dx}=a\theta\sin\theta\times\frac{1}{a\theta\cos\theta}$

$\Rightarrow \frac{dy}{dx}=\tan\theta$

55.

(c) $f''(e^x)e^{2x}+f'(e^x)e^x$

Explanation: Let $y = f(e^x)$, then

$y_1 = f'(e^x)e^x$

$y_2 = f''(e^x)e^xe^x + f'(e^x)e^x$

$= e^{2x}f''(e^x) + f'(e^x)e^x$

56. **(a)** continuous $\forall$ x $\in$ R and not differentiable at x = 0

Explanation: f(x) = x - |x| = g(x) + h(x)

where y(x) = x, h(x) = -|x|
As g(x) and h(x) are both continuous $\forall$ x $\in$ R
$\therefore$ f(x) is continuous $\forall$ x $\in$ R
And g(x) is differentiable $\forall$ x $\in$ R
but h(x) is not differentiable at x = 0
$\therefore$ f(x) = g(x) + h(x) is not differentiable at x = 0

57.

(d) continuous at x = 0

Explanation: Given f(x) = $\sin^{-1}$(cos x),
Checking differentiability and continuity,
LHL at x = 0,
$\lim_{\mathbf{x}\to 0^-} \mathbf{f}(\mathbf{x}) = \lim_{\mathbf{h}\to 0} \mathbf{f}(0-\mathbf{h}) = \lim_{\mathbf{h}\to 0} \sin^{-1}(\cos(0-\mathbf{h})) = \lim_{\mathbf{h}\to 0} \sin^{-1}(\cos(-\mathbf{h})) = \sin^{-1} 1 = \frac{\pi}{2}$
RHL at x =0,
$\lim_{\mathbf{x}\to 0^+} \mathbf{f}(\mathbf{x}) = \lim_{\mathbf{h}\to 0} \mathbf{f}(0+\mathbf{h}) = \lim_{\mathbf{h}\to 0} \sin^{-1}(\cos(0+\mathbf{h})) = \lim_{\mathbf{h}\to 0} \sin^{-1}(\cos(\mathbf{h})) = \sin^{-1} 1 = \frac{\pi}{2}$
And $f(0) = \frac{\pi}{2}$
Hence, f(x) is continuous at x =0.
LHD at x = 0,
$\lim_{x\to 0^-} \frac{f(x)-f(0)}{x-0} = \lim_{h\to 0} \frac{f(0-h)-f(0)}{0-h-0}$
$= \lim_{h\to 0} \frac{\sin^{-1}(\cos(0-h))-\left(\frac{\pi}{2}\right)}{-h} = 1$
RHD at x =0,
$\lim_{x\to 0^+} \frac{f(x)-f(0)}{x-0} = \lim_{h\to 0} \frac{f(0+h)-f(0)}{0+h-0}$
$= \lim_{h\to 0} \frac{\sin^{-1}(\cos(0+h))-\left(\frac{\pi}{2}\right)}{h} = -1$
$\because$ LHD $\neq$ RHD
$\therefore$ f(x) is not differentiable at x =0.

58. **(a)** $(2/3)^{1/2}$

Explanation: $f(x) = \cot^{-1}(\sqrt{\cos 2x})$
$f'(x) = \frac{-1}{1+\cos 2x} \times \frac{1}{2\sqrt{\cos 2x}} \times -2\sin 2x$
$f'(x) = \frac{-1}{2\cos^2 x} \times \frac{1}{2\sqrt{\cos 2x}} \times -2\sin 2x$
$f'\left(\frac{\pi}{6}\right) = \frac{1}{2\cos^2 \frac{\pi}{6}} \times \frac{1}{2\sqrt{\cos \frac{\pi}{3}}} \times 2\sin \frac{\pi}{3}$
$f'\left(\frac{\pi}{6}\right) = \frac{1}{2\times \frac{3}{4}} \times \frac{1}{2\sqrt{\frac{1}{2}}} \times 2\frac{\sqrt{3}}{2} = \left(\frac{2}{3}\right)^{\frac{1}{2}}$

59. **(a)** $\frac{11}{4}$

Explanation: $\frac{11}{4}$

60.

(c) $\frac{\log x}{(1+\log x)^2}$

Explanation: $x^y = e^{x-y}$
Taking log on both sides,
$\log x^y = \log e^{x-y}$
y log x = x - y
y log x + y = x
$y = \frac{x}{\log x+1}$
Differentiate with respect to x,
$\frac{dy}{dx} = \frac{(\log x+1)-x\times\frac{1}{x}}{(\log x+1)^2}$

$\frac{dy}{dx}=\frac{(\log x+1)-1}{(\log x+1)^2}$
$\frac{dy}{dx}=\frac{\log x}{(\log x+1)^2}$

61. **(a)** $\frac{n^2+1}{n}$
Explanation: $\frac{n^2+1}{n}$

62. **(a)** $\frac{1}{2}$
Explanation: Given that y = $\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$

Using 1 - cos x = 2 $\sin^2\frac{x}{2}$ and 1 + cos x = 2 $\cos^2\frac{x}{2}$, we obtain

$y=\tan^{-1}\sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}=\tan^{-1}\tan\left(\frac{x}{2}\right)=\frac{x}{2}$

Differentiating with respect to x, we
$\frac{dy}{dx}=\frac{1}{2}$

63.
(d) $\frac{\sqrt{e^x-1}}{2}$

Explanation: Given, x = log (1 + t^2) and y = t - $\tan^{-1}$ t
On differentiating both sides w.r.t.x, we get
$\frac{dx}{dt}$ = $\frac{1}{1+t^2}(2t)$ and $\frac{dy}{dt}$ = 1 - $\frac{1}{1+t^2}$ = $\frac{t^2}{1+t^2}$

$\therefore\frac{dy}{dx}$ = $\frac{\frac{t^2}{(1+t^2)}}{\frac{2t}{(1+t^2)}}$ = $\frac{t}{2}$ ...(i)

Also, x = log (1 + t^2)

$\Rightarrow t^2 = e^x-1$ ...(ii)
From Eqs. (i) and (ii), we get
$\frac{dy}{dx}=\frac{\sqrt{e^x-1}}{2}$

64.
(d) f'(x) exists for all x

Explanation: We have f(x) = $\frac{\sin 4\pi[\pi^2 x]}{7+[x]^2}$

We know that $[\pi^2$ x] is an integer for every x

$\therefore 4\pi[\pi^2$x] is an integral multiple of π

$\therefore$ sin $4\pi[\pi^2$x] = 0 and 7 + $[x]^2 \neq 0\ \forall$ x

$\therefore$ f(x) = 0 $\forall$ x

$\Rightarrow$ f(x) is a constant function of f'(x), f'(x), f''(x), ...f^n(x) exists $\forall$ x.

65.
(d) $\frac{\pi}{4}+\frac{1}{2}$

Explanation: $f'(x)=\frac{d}{dx}(x\tan^{-1}x)=\frac{x}{1+x^2}+\tan^{-1}x\ \Rightarrow f'(1)=\frac{1}{1+1}+\tan^{-1}1=\frac{1}{2}+\frac{\pi}{4}$

66. **(a)** $\frac{2\log x-3}{x^3}$

Explanation: $\frac{d}{dx}\left(\frac{\log x}{x}\right)=\frac{x.\frac{1}{x}-\log x.1}{x^2}=\frac{1-\log x}{x^2}$
$\Rightarrow\frac{d}{dx}\left(\frac{1-\log x}{x^2}\right)$
$=\frac{x^2.(-\frac{1}{x})-(1-\log x).2x}{x^4}=\frac{-x-2x+2x\log x}{x^4}=\frac{x(2\log x-3)}{x^4}=\frac{2\log x-3}{x^3}$

67.
(c) y = x

Explanation: y = x + e^{-x} sin x = x + $\frac{\sin x}{e^x}$
$\because\lim\limits_{x\to\infty}\frac{\sin x}{e^x}$ = 0
As -1 $\leq$ sin x $\leq$ 1

$\therefore$ -e^{-x} $\leq$ e^{-x} sin x $\leq$ e^{-x}

$\lim\limits_{x \to \infty} \pm$ e^{-x} = 0

$\therefore \lim\limits_{x \to \infty}$ e^{-x} sin x = 0

$\therefore$ Line y = x is oblique asymptote to the given curve.

68.

(c) 1

Explanation: u = sin^{-1} $\left(\frac{2x}{1+x^2}\right) = 2\tan^{-1}x \Rightarrow \frac{du}{dx} = \frac{2}{1+x^2}$

v = tan^{-1} $\left(\frac{2x}{1+x^2}\right) = 2\tan^{-1}x \Rightarrow \frac{du}{dx} = \frac{2}{1+x^2}$

$\therefore \frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{2/1+x^2}{2/1+x^2} = 1$

69. **(a)** continuous and differentiable at x = 0.

Explanation: continuous and differentiable at x = 0.

Given f(x) = x|x|

function can be written as,

$$f(x) = \begin{cases} x(x) & \text{if } x \geq 0 \\ x(-x) & \text{if } x < 0 \end{cases}$$

$$f(x) = \begin{cases} x^2 & \text{if } x \geq 0 \\ -x^2 & \text{if } x < 0 \end{cases}$$

Now,

$Rf(0) = \lim_{x \to 0^-} (-x^2)$ = 0

$Lf(0) = \lim_{x \to 0^+} (x^2)$ = 0

f(0) = 0

$\because Rf(0) = Lf(0) = f(0)$

so f(x) is continuous at x=0

Now

$Rf'(0) = \lim_{x \to o^-} \frac{f(x)-f(0)}{x-0}$

$= \lim_{x \to o^-} \frac{(-x^2)-0}{x-0} = 0$

$Lf'(0) = \lim_{x \to o^+} \frac{f(x)-f(0)}{x-0}$

$= \lim_{x \to o^+} \frac{x^2-0}{x-0} = 0$

$\because Rf'(0) = Lf'(x)$

so f(x) is continuous at x=0

hence f(x) is continuous and differentiable at x = 0.

70. **(a)** 0

Explanation: F(x) = $\begin{vmatrix} f(x) & g(x) & h(x) \\ f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix}$

Where f(x), g(x), h(x) are polynomial of degree 2.

$\therefore$ f'''(x) = 0 g'''(x) = h'''(x)

F' (x) = $\begin{vmatrix} f'(x) & g'(x) & h'(x) \\ f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix}$ + $\begin{vmatrix} f(x) & g(x) & h(x) \\ f''(x) & g''(x) & h''(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix}$ + $\begin{vmatrix} f(x) & g(x) & h(x) \\ f'(x) & g'(x) & h'(x) \\ f'''(x) & g'''(x) & h'''(x) \end{vmatrix}$

= 0 + 0 + 0 = 0

71.

(d) 6

Explanation: $\lim\limits_{x \to 0} \frac{\sin 3x}{x} = \frac{k}{2}$

$\lim\limits_{x \to 0} \frac{\sin 3x}{3x} \times 3 = \frac{k}{2}$

$3 = \frac{k}{2}$

k = 6

72. **(a)** $\frac{1}{(1+x^2)}$

Explanation: Given that $y=\cot^{-1}\left(\frac{1-x}{1+x}\right)$

Let x = $\tan\theta \Rightarrow \theta=\tan^{-1}x$ and using $\cot^{-1}x=\frac{\pi}{2}-\tan^{-1}x$

Hence, $y=\frac{\pi}{2}-\tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$

Using $\tan\left(\frac{\pi}{4}-x\right)=\frac{1-\tan x}{1+\tan x}$, we obtain

$y=\frac{\pi}{2}-\tan^{-1}\tan\left(\frac{\pi}{4}-\theta\right)=\frac{\pi}{2}-\left(\frac{\pi}{4}-\theta\right)=\frac{\pi}{4}+\theta=\frac{\pi}{4}+\tan^{-1}x$

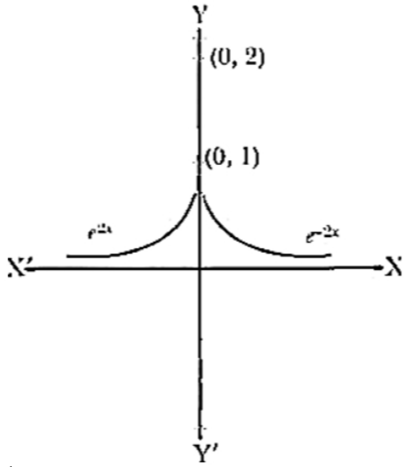
Differentiating with respect to x, we obtain

$\frac{dy}{dx}=\frac{1}{1+x^2}$

73.

(c) continuous $\forall$ x $\in$ R and non differentiable at x = ± 1

Explanation:



Given g(x) = $\begin{cases} e^{2x}, & x<0 \\ e^{-2x}, & x\geq 0\end{cases}$

g'(x) = $\begin{cases} 2e^{2x}, & x<0 \\ -2e^{-2x}, & x\geq 0\end{cases}$

$\therefore$ LHD at x = 0, g'(0) = $2e^{2\times 0} = 2e^0 = 2$

RHD at x = 0, g'(0) = $-2e^0 = -2\times 1 = -2$

As LHD $\neq$ RHD at x = 0

$\therefore$ g(x) is not differentiable at x = 0

Again RHL = $\lim\limits_{x\to 0^+}$ g(x) = $\lim\limits_{x\to 0^+}$ $e^{-2x} = e^0 = 1$

LHL = $\lim\limits_{x\to 0^-}$ g(x) = $\lim\limits_{x\to 0^-}$ $e^{2x} = e^0 = 1$

g(0) = e^0 = 1

As LHL = RHL = f(0)

$\therefore$ g(x) is continuous $\forall$ x $\in$ R

74.

(d) none of these

Explanation: Given that $f(x)=\left\{\begin{array}{c}\frac{-1}{x}, x\leq -1\\ ax^2+b, -1<x<1\\ \frac{1}{x}, x\geq 1\end{array}\right\}$

$\because$ f(x) is continuous and differentiable at any point, consider x =1.

$\lim\limits_{x\to 1}\frac{1}{x}=\lim_{x\to 1}ax^2+b$

$\Rightarrow$a + b = 1

Also,

$\Rightarrow \lim\limits_{x\to 1^-}\frac{f(x)-f(1)}{x-1}=\lim\limits_{x\to 1^+}\frac{f(x)-f(1)}{x-1}$

$\Rightarrow \lim\limits_{x\to 1}\frac{ax^2-a}{x-1}=\lim\frac{1-x}{x(x-1)}$

$\Rightarrow \lim_{x \to 1} a(x+1) = \lim_{x \to 1}(-x)$
$\Rightarrow a = \frac{-1}{2}$
Putting above value in a + b = 1,we get
$b = \frac{3}{2}$.
Which is the required value of a and b.

75.

(d) -cot y $cosec^2$ y

Explanation: Given, y = $\cos^{-1}$ x
⇒ x = cos y
On differentiating both sides w.r.t. x, we get
$\frac{dx}{dy} = -\sin y \Rightarrow \frac{dy}{dx}$ = -cosec y ...(i)
Again, differentiating both sides w.r.t. x, we get
$\frac{d^2y}{dx^2} = \frac{d}{dx}(-\operatorname{cosec} y) = -(-\operatorname{cosec} y \cot y)\frac{dy}{dx}$
= cosec y cot y(-cosec y)
= -cot y · $cosec^2$ y [from Eq. (i)]