# Solution

## CET25M6

## Class 12 - Mathematics

1.

**(c)** 2abc

**Explanation:** S = $b^2x + c^2y$ and $xy = a^2$

$\Rightarrow$ S = $b^2x + \frac{a^2c^2}{x}$

$\therefore \frac{dS}{dx} = b^2 - \frac{c^2a^2}{x^2} = 0$

$\Rightarrow b^2 - \frac{c^2a^2}{x^2} = 0 \Rightarrow x^2 = \frac{c^2a^2}{b^2} \Rightarrow x = \pm\frac{ca}{b}$

$\frac{d^2S}{dx^2} = \frac{2c^2a^2}{x^3}$

$\therefore \frac{d^2S}{dx^2}\Big|_{x=\frac{ca}{b}} = \frac{2c^2a^2}{c^3a^3} = \frac{2b^3}{ca} > 0$

$\therefore$ Minimum value of S = $b^2 \times \frac{ca}{b} + \frac{c^2a^2}{\frac{ca}{b}}$ = abc + abc = 2abc

2.

**(c)** x = -2

**Explanation:** Given function, $f(x) = x^3 + 2x^2 - 4x + 6$

and $f'(x) = 3x^2 + 4x - 4$

Now, for maximum or minimum of f(x), put f'(x) = 0

$\Rightarrow 3x^2 + 4x - 4 = 0$

$\Rightarrow (x + 2)(3x - 2) = 0$

$\Rightarrow x = -2, \frac{2}{3}$

Now, f''(x) = 6x + 4,

At x = -2,

f''(-2) = 6(-2) + 4 = -12 + 4

= -8 < 0 [maximum]

So, maximum value of the function f(x) exists at x = -2

3. **(a)** 2

**Explanation:** Here ,it is given the function $f(x) = e^x + e^{-x}$

$\Rightarrow f(x) = e^x + \frac{1}{e^x}$

$\Rightarrow f(x) = \frac{e^{2x}+1}{e^x}$

f(x) is always increasing at x = 0 it has the least value

$\Rightarrow f(x) = \frac{1+1}{1} = 2$

$\therefore$ The least value is 2

4.

**(b)** 0 < x < 1

**Explanation:** 0 < x < 1

5.

**(d)** R

**Explanation:** R

6.

**(c)** $\frac{1}{6}$

**Explanation:** f(x) = $\frac{x}{4+x+x^2}$

$\Rightarrow$ f'(x) = $\frac{4-x^2}{(4+x+x^2)^2}$

For a local maxima or minima,

f'(x) = 0

$\frac{4-x^2}{\left(4+x+x^2\right)^2} = 0$

$\Rightarrow x = \pm 2 \in [-1, 1]$

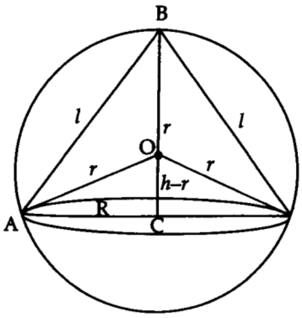
$f(1) = \frac{1}{6} > 0$

$f(-1) = \frac{-1}{4} < 0$

$\Rightarrow \frac{1}{6}$ is the maximum value.

7.

**(d)** $2\pi^2 r(2rh^2 - h^3)$

**Explanation:** 


Here, CSA of cone = $\pi Rl$

Radius of sphere = r

height of cone = h

In $\triangle AOC$,

$AO^2 = AC^2 + OC^2$

$\Rightarrow r^2 = R^2 + (h - r)^2$

$\Rightarrow R^2 = 2hr - h^2$

$\therefore$ Radius of cone, $R = \sqrt{2hr - h^2}$ ...(i)

In $\triangle ABC$,

$AB^2 = AC^2 + BC^2$

$\Rightarrow l^2 = R^2 + h^2$

$\Rightarrow l^2 = 2hr - h^2 + h^2$

$\therefore$ slant height $= \sqrt{2hr}$ ...(ii)

CSA of cone = $\pi Rl$

$= \pi\sqrt{2hr - h^2}\sqrt{2hr}$

$(\text{CSA of cone})^2 = \pi^2(2hr - h^2)(2hr)$

$= 2\pi^2 hr(2hr - h^2)$

$= 2\pi^2 r(2rh^2 - h^3)$

8.

**(d)** none of these

**Explanation:** Let f(x) = |x| is not differential.

$\because f'(0) \neq 0$ but f(x) has a minimum at x = 0.

9.

**(d)** (-1, 1)

**Explanation:** We have, $\Rightarrow f(x) = \frac{x}{x^2+1}$

$\Rightarrow f'(x) = \frac{x^2 - 2x^2 + 1}{x^2+1}$

$\Rightarrow f'(x) = -\frac{x^2 - 1}{x^2+1}$

$\Rightarrow$ for critical points f'(x) = 0

when f'(x) = 0

We get x = 1 or x = -1

When we plot them on number line as f'(x) is multiplied by -ve sign we get

For x > 1 function is decreasing

For x < -1 function is decreasing

But between -1 to 1 function is increasing

$\therefore$ Function is increasing in ( -1, 1)

10.

**(d)** monotonic function

**Explanation:** monotonic function

11. **(a)** a = $\frac{1}{2}$

**Explanation:** a = $\frac{1}{2}$

12.

**(c)** 0 < x < 1 and x > 2

**Explanation:** Given function is

y = $[x(x - 2)]^2 = [x^2 - 2x]^2$

On differentiating w.r.t. x, we get

$\frac{dy}{dx} = 2(x^2 - 2x) \frac{d}{dx}(x^2 - 2x)$

$= 2(x^2 - 2x(2x - 2)$

= 4x(x - 2)(x - 1)

On putting $\frac{dy}{dx}$ = 0, we get

x = 0, 1 and 2

which divides real line in disjoint intervals ($-\infty$, 0), (0, 1), (1, 2) and (2, $\infty$).


| Intervals | Sign of f'(x) | Nature of f(x) |
|---|---|---|
| $(-\infty, 0)$ | (-)(-)(-) = -ve | Strictly decreasing |
| (0, 1) | (+)(-)(-) = +ve | Strictly increasing |
| (1, 2) | (+)(-)(+) = -ve | Strictly decreasing |
| $(2, \infty)$ | (+)(+)(+) = +ve | Strictly increasing |

Therefor, y in increasing in (0, 1) and (2, $\infty$).

13.

**(b)** ₹ 10

**Explanation:** The marginal cost is the rate of change of cost w.r.t. the no. of units produced.

i.e., $\text{Marginal cost }(MC) = \frac{dP(x)}{dt}$

$= \frac{d}{dt}\left(0.4x^2 + 2x - 10\right)$

= 0.8x + 2

$\therefore$ Marginal cost $(MC)|_{\text{at } x=8} = 0.8 \times 10 + 2$ = ₹10

14. **(a)** Minimum at x = $\frac{\pi}{2}$

**Explanation:** f(x) = 1 + 2 sinx + $3\cos^2$ x

$\Rightarrow$ f'(x) = 2 cosx - 6 cosx sinx

$\Rightarrow$ f'(x) = 2 cosx - 3 sin2x

to find minima or maxima of the function

2 cosx - 6 cosx sinx = 0

2 cos x (1 - 3sinx) = 0

$\Rightarrow$ cos x = 0 or sinx = $\frac{1}{3}$

x = $\frac{\pi}{2}$ or x = $\sin x^{-1}\left(\frac{1}{3}\right)$

f''(x) = - 2 sinx - 6 cos2x

$f''\left(\frac{\pi}{2}\right)$ = 4 > 0

$\Rightarrow$ x = $\frac{\pi}{2}$ is a local minima.

$f''\left(\sin^{-1}\left(\frac{1}{3}\right)\right) = -\left(\frac{2}{3}+4\sqrt{2}\right) < 0$

function has maxima at $x = \sin^{-1}\left(\frac{1}{3}\right)$

15.

**(c)** increasing

**Explanation:** increasing

16.

**(b)** only one minima

**Explanation:** Given, $f(x) = |x| = \begin{matrix} -x, & x<0 \\ x, x>0 \end{matrix}$

$\Rightarrow f'(x) = -1$ when x< 0 and 1 when x > 0

But, we have f'(x) does not exist at x = 0, hence we have x = 0 is a critical point

At x = 0, we get f(0) = 0

For any other value of x, we have f(x) > 0 , hence f(x) has a minimum at x= 0.

17.

**(c)** 4 cm/s

**Explanation:** Let the side of the square be x.

Area of square, $A = x^2$

Differentiating w.r.t. 't', we get

$\frac{dA}{dt} = 2x\frac{dx}{dt}$

Since, given $\frac{dA}{dt} = 40 \text{ cm}^2/\text{s}$

$\therefore 40 = 2x\frac{dx}{dt}$

$\Rightarrow \frac{dx}{dt} = \frac{20}{x}$

Thus, $\left.\frac{dx}{dt}\right|_{x=5} = \frac{20}{5} = 4 \text{ cm/s}$

18.

**(d)** $1, \frac{1}{3}$

**Explanation:** Given, $f(x) = x^3 - 2x^2 + x + 1$

$\therefore f'(x) = 3x^2 - 4x + 1$

For critical points, put f'(x) = 0

$\therefore 3x^2 - 4x + 1 = 0$

$\Rightarrow 3x^2 - 3x - x + 1 = 0$

$\Rightarrow 3x(x - 1) - 1(x - 1) = 0$

$\Rightarrow (3x - 1)(x - 1) = 0 \Rightarrow x = 1, \frac{1}{3}$

19.

**(c)** $\sqrt{a^2 - b^2}$

**Explanation:** Let $y = a \sec\theta - b \tan\theta$

$\frac{dy}{d\theta} = a \sec\theta \tan\theta - b \sec^2\theta$

For minimum value of y

$\frac{dy}{d\theta} = 0 = a \sec\theta \tan\theta - b \sec^2\theta$

$\Rightarrow \sin\theta = \frac{b}{a}$

Hence, $y = \frac{a - b\sin\theta}{\cos\theta} = \frac{a^2-b^2}{\sqrt{a^2-b^2}} = \sqrt{a^2 - b^2}$

20.

**(c)** Decreasing on R

**Explanation:** Given, $f'(x) = -x^3 + 3x^2 - 3x + 4$

$f'(x) = -3x^2 + 6x - 3$

$f'(x) = -3(x^2 - 2x + 1)$

$f'(x) = -3(x - 1)^2$

As f'(x) has -ve sign before 3

$\Rightarrow$ f'(x) is decreasing over R.

21. **(a)** decreases on [0, a]

**Explanation:** decreases on [0, a]

22. **(a)** f(x) is an increasing function

**Explanation:** f'(x) = $\frac{\sin x - x\cos x}{\sin^2 x}$, g'(x) = $\frac{\tan x - x\sec^2 x}{\tan^2 x}$

Now $\frac{d}{dx}$(sin x - x cos x) = cos x + x sin x - cos x

= x sin x > 0 for 0 $\leq$ x < 1

$\therefore$ sin x - x cos x is an increasing function.

But at x = 0, x sin x is 0

$\therefore$ In 0 < x $\leq$ 1, sin x - x cos x > 0

$\therefore$ f'(x) > 0 for 0 < x $\leq$ 1

So f(x) is increasing in the interval 0 < x $\leq$ 1

Again $\frac{d}{dx}$(tan x - x $\sec^2$x) = $\sec^2$x - 2x $\sec^2$x tan x - $\sec^2$x

= -2x $\sec^2$x tan x < 0 for 0 $\leq x \leq$ 1

$\therefore$ g(x) is decreasing in 0 < x $\leq$ 1

23. **(a)** point of inflexion at x = 0

**Explanation:** Given, f(x) = x|x| $= \begin{cases} -x^2, & x<0 \\ x^2, & x>0 \end{cases}$

$\Rightarrow f'(x) = -2x$ when x < 0 and 2x when x > 0 $f'(x) = 0 \Rightarrow x = 0$

Hence f(x) has a point of inflexion at x = 0.

But, x = 0 is not a local extreme as we cannot find an interval I around x = 0 such that $f(0) \geq f(x)\ or\ f(0) \leq f(x) \quad \forall x \in I$

24. **(a)** strictly decreasing

**Explanation:** Given function is f(x) = $\frac{5}{x}$ - 9, x $\neq$ 0

$\Rightarrow$ f(x) = $5x^{-1}$ - 9

On differentiating both sides w.r.t. x, we get

f'(x) = 5(-1) $x^{-2}$ - 0 = $\frac{-5}{x^2}$ < 0, $\forall$ x $\in$ R - {0}

$\therefore$ f(x) is strictly decreasing for x $\in$ R, (x $\neq$ 0).

25.

**(c)** $\frac{\pi}{6}$

**Explanation:** Let y = sin $\theta\cdot$ sin $\phi$ = sin $\theta\cdot$ sin $(\frac{\pi}{3} - \theta)$

For maximum value of y.

$\frac{dy}{dx}$ = 0 = cos $\theta\cdot$ sin $(\frac{\pi}{3} - \theta)$ − sin $\theta\cdot$ cos $(\frac{\pi}{3} - \theta)$ = sin $(2\theta - \frac{\pi}{3})$

$\Rightarrow \theta = \frac{\pi}{6}$

26.

**(b)** 15

**Explanation:** y = $-x^3 + 3x^2 + 12x - 5$

$\frac{dy}{dx}$ = $-3x^3 + 6x + 12$ = f(x)

$\frac{d^2y}{dx^2}$ = -6x + 6

$\frac{d^2y}{dx^2}$ = 0

-6x + 6 = 0

x = 1

$\frac{d^3y}{dx^2}$ = -6 < 0 ,maximum

i.e, $\frac{dy}{dx}$ or slope of y is maximum at x = 1

Slope = f(x) = $-3(1)^2 + 6(1) + 12$

= -3 + 6 + 12

= 15

27.

**(b)** $\lambda > 2$

**Explanation:** $\lambda > 2$

28. **(a)** $\frac{a+b+c}{3}$

**Explanation:** f(x) = $(x - a)^2 + (x - b)^2 + (x - c)^2$

⇒f'(x) = 2(x - a) + 2(x - b) + 2(x - c)

to find minima or maxima

f'(x) = 0

2(x - a) + 2(x - b) + 2(x - c) = 0

⇒ x = $\frac{a+b+c}{3}$

f''(x) = 6 > 0

function has minima at x = $\frac{a+b+c}{3}$.

29.

**(c)** (-∞, -1)

**Explanation:** f(x) = sin x - ax + b

⇒ f'(x) = cos x - a

For increasing function

f'(x) $\geq$ 0

cos x - a $\geq$ 0 ⇒ cos x $\geq$ a

i.e. a $\leq$ cos x a $\leq$ min (cos x) = -1

∴ a ∈ (-∞, -1)

30. **(a)** $e^{\frac{-1}{e}}$

**Explanation:** Here,it is given the function f(x) = $x^x$

⇒ Keeping f'(x)= $x^x(1 + logx)$ = 0

We get

⇒ x = 0 or $x = \frac{1}{e}$

⇒ $f''(x) = x^x(1 + logx + \frac{1}{x})$

When x is greater than zero. Then $f''(x) \leq 0$

We get a maximum value as the function will be negative

Thus,

f(x) = $x^x$

$f(e) = \left(\frac{1}{e}\right)^{1/e} = e^{-\frac{1}{e}}$

31.

**(c)** Increasing (-∞, -1)

Decreasing (-1, ∞)

**Explanation:** f(x) = $-x^2$ - 2x + 15

f'(x) = -2x - 2 = -2(x + 1) > 0

if x < -1 i.e., in (-∞, -1)

f'(x) < 0 if x > - 1 i.e., in (-1, ∞)

Hence f(x) is increasing in (-∞, -1) and decreasing in (-1, ∞).

32. **(a)** 2

**Explanation:** Given xy = 1. To find minimum value of x + y

⇒ y = $\frac{1}{x}$

f(x) = x + $\frac{1}{x}$

⇒ f'(x) = 1 - $\frac{1}{x^2}$

To find local maxima or minima we have

f'(x) = 0

1 - $\frac{1}{x^2}$ = 0

⇒ x = ±1 ⇒ y =±1

But given that $x > 0 \Rightarrow x = 1, y = 1$
$f''(x) = \frac{2}{x^3}$
$f''(1) = 2 > 0$
function has minima at x = 1
$f(1) = 2$.

33.

**(d)** $\tan^{-1} \frac{3}{5}$

**Explanation:** $\tan^{-1} \frac{3}{5}$

34. **(a)** $(1, \infty)$

**Explanation:** Given, function
$\Rightarrow f(x) = (x + 1)^3 . (x - 3)^3$
$\Rightarrow f'(x) = 3(x + 1)^2 (x - 3)^3 + 3(x - 3)^3 (x + 1)^3$
Put $f'(x) = 0$
$\Rightarrow 3(x + 1)^2 (x - 3)^3 = -3(x - 3)^2 (x + 1)^3$
$\Rightarrow x - 3 = -(x + 1)$
$\Rightarrow 2x = 2$
$\Rightarrow x = 1$
When x > 1 the function is increasing
x < 1 function is decreasing
Therefore f(x) is increasing in $(1, \infty)$.

35. **(a)** f(x) is strictly increasing on R

**Explanation:** Let $x_1$ and $x_2$ be any two numbers in R. Then,
$x_1 < x_2 \Rightarrow 9x_1 < 9x_2 \Rightarrow 9x_1 - 5 < 9x_2 - 5$
$\Rightarrow f(x_1) < f(x_2)$
Thus, f is strictly increasing on R.

36.

**(c)** 252 $cm^2/s$

**Explanation:** Let the edge of the cube be a.
The rate of change of edge of the cube is given by $\frac{dA}{dt}$
The area of the cube is $A = 6a^2$
Differentiating w.r.t. 't', we get
$\frac{dA}{dt} = 12a\frac{da}{dt}$
$\therefore \frac{dA}{dt} = 12a \times 7 = 84a$
Thus, $\left.\frac{dA}{dt}\right|_{a=3} = 84 \times 3 = 252 \text{ cm}^2/\text{s}$

37. **(a)** $\frac{\pi}{6}$

**Explanation:** $f(x) = \sin x + \sqrt{3} \cos x$
$\Rightarrow f'(x) = \cos x - \sqrt{3} \sin x$
for maxima or minima
$f'(x) = 0$
$\cos x - \sqrt{3} \sin x = 0$
$\Rightarrow \tan x = \frac{1}{\sqrt{3}} \Rightarrow x = \frac{\pi}{6}$
$f''(x) = - \sin x - \sqrt{3} \cos x$
$\Rightarrow f''\left(\frac{\pi}{6}\right) = - \sin\frac{\pi}{6} - \sqrt{3} \cos \frac{\pi}{6} = \frac{-1-\sqrt{3}}{2} < 0$
function has local maxima at $x = \frac{\pi}{6}$

38.

**(b)** $\frac{4}{3}$

**Explanation:** $f(x) = 4x^2 + 2x + 1$
$\Rightarrow f'(x) = 8x + 2$

For local minima or maxima we have

f'(x) = 8x + 2 = 0

⇒ x = $\frac{-1}{4}$

f''(x) = 8 > 0

⇒ function has maxima at x = $\frac{-1}{4}$

$f\left(\frac{-1}{4}\right)=\frac{4}{3}$

39. **(a)** $\frac{1}{2}$

**Explanation:** Let, the numbers whose sum is 8 are 8, 8 - x.

Given f(x) = $\frac{1}{x}+\frac{1}{8-x}$

⇒ f'(x) = $\frac{-1}{x^2}+\frac{1}{(8-x)^2}$

to find minima or maxima

f'(x) = 0

⇒ $\frac{-1}{x^2}+\frac{1}{(8-x)^2}$ = 0

⇒ x = 4

f''(x) = $\frac{2}{x^3}-\frac{2}{(8-x)^3}$

⇒ f''(4) = $\frac{2}{4^3}-\frac{2}{(8-4)^3}$ = 0

Minimum value of the sum of their reciprocals = $\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$

40. **(a)** x ∈ R

**Explanation:** x ∈ R

41.

**(b)** none of these

**Explanation:** f(x) = x + $\frac{1}{x}$

⇒ f'(x) = 1 -$\frac{1}{x^2}$

For minimum or maximum value of the function

f'(x) = 0

⇒ 1 - $\frac{1}{x^2}$ = 0

⇒ x = $\pm 1$

f''(x) = $\frac{2}{x^3}$

⇒ f''(x) = 1 > 0 ⇒ function has minima at x = 1.

f''(-1) = -1 > 0 ⇒ function has minima at x = -1.

42.

**(c)** always increases

**Explanation:** We have, f(x) = tan x - x

∴ f'(x) = $\sec^2$x - 1

⇒ f'(x) $\geq$ 0, ∀ x ∈ R

So, f(x) always increases.

43.

**(d)** $x=\frac{1}{e}$

**Explanation:** Consider f(x) = y =$x^x$

Then, log y = log $x^x$ = x.logx

$\Rightarrow f'(x)=x^x(1+\log x)$

$\Rightarrow (1+\log x)=0 \ldots\ldots(\because x^x \neq 0)$

$\Rightarrow \log x=-1 \Rightarrow x=e^{-1}$

44. **(a)** -1 < x < -3

**Explanation:** Here,it is given that function,

f(x) = $x^3$ + $6x^2$ + 9x + 3

$f'(x) = 3x^2 + 12x + 9 = 0$

$f'(x) = 3(x^2 + 4x + 3) = 0$

$f'(x) = 3(x + 1)(x + 3) = 0$

$x = -1$ or $x = -3$

for $x > -1$ f(x) is increasing

for $x < -3$ f(x) is increasing

But for $-1 < x < -3$ it is decreasing

45. **(a)** $\left(0, \frac{\pi}{4}\right)$

**Explanation:** Given function is f(x) = sin x + cos x

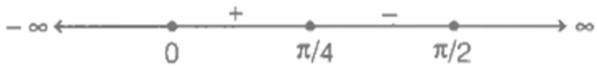
On differentiating both sides w.r.t. x, we get

$f'(x) = \cos x - \sin x$

Put $f'(x) = 0 \Rightarrow \cos x - \sin x = 0$

$\Rightarrow \tan x = 1 \Rightarrow x = \frac{\pi}{4} \in (0, \frac{\pi}{2})$

Now, $\frac{\pi}{4}$ divides the interval $(0, \frac{\pi}{2})$ into two sub intervals $(0, \frac{\pi}{4})$ and $(\frac{\pi}{4}, \frac{\pi}{2})$.




In interval $(0, \frac{\pi}{4})$, cos x > sin x

$\therefore \cos x - \sin x > 0 \Rightarrow f'(x) > 0$

So, f(x) is strictly increasing function in $(0, \frac{\pi}{4})$.

In interval $(\frac{\pi}{4}, \frac{\pi}{2})$, cos x < sin x

$\therefore \cos x - \sin x < 0 \Rightarrow f'(x) < 0$

So, f(x) is strictly decreasing function in $(\frac{\pi}{4}, \frac{\pi}{2})$.

46. **(a)** none of these

**Explanation:** Given: $f(x) = x^3 - 6x^2 + 9x$

$\Rightarrow f'(x) = 3x^2 - 12x + 9$

For a local maxima or a local minima, we must have

$f'(x) = 0$

$\Rightarrow 3x^2 - 12x + 9 = 0$

$\Rightarrow x^2 - 4x + 3 = 0$

$\Rightarrow (x - 1)(x - 3) = 0$

$\Rightarrow x = 1, 3$

Now,

$f(0) = 0^3 - 6(1)^2 + 9(1) + 9(1) = 1 - 6 + 9 = 4$

$f(1) = 1^3 - 6(1)2 + 9(1) = 1 - 6 + 9 = 4$

$f(3) = 3^3 - 6(3)^3 + 9(3) = 27 - 54 + 27 = 0$

$f(6) = 6^3 - 6(6)^2 + 9(6) = 216 - 216 + 54 = 54$

The least and greatest values of $f(x) = x^3 - 6x^2 + 9x$ in [0, 6] are 0 and 54, respectively.

47.

**(c)** a = 11, b = -6

**Explanation:** a = 11, b = -6

48.

**(c)** 1

**Explanation:** $f(x) = \cos x + \cos(\sqrt{2}x)$

$\therefore f(x) = 2 \cos \frac{\sqrt{2}+1}{2}x \cos \frac{\sqrt{2}-1}{2} x \leq 2$

and it is 2 when $\cos \frac{\sqrt{2}+1}{2}x$ and $\cos \frac{\sqrt{2}-1}{2}$ are both equal to 1 for a value of x. This is possible only when x = 0.

49. **(a)** $a = 2, b = -\frac{1}{2}$

**Explanation:** Let $f(x) = a\log x + bx^2 + x$

$f'(x) = a.\frac{1}{x} + 2bx + 1$

For maximum and minimum value of f(x) we have f'(x) = 0

Therefore, at x = -1 and x = 2 we have $2bx^2 + x + a = 0$

i.e, a + 2b = 1....(i) and a + 8b = 2....(ii)

(ii) - (i ) gives b = $-\frac{1}{2}$

Now , from (i) we get a = 2

$\Rightarrow$ a = 2, b = $-\frac{1}{2}$

50.

**(d)** (0 , $\frac{1}{e}$)

**Explanation:** (0 , $\frac{1}{e}$)

Let y = $x^x$

$\Rightarrow \log(y)$ = xlogx

$\Rightarrow \frac{1}{y} \times \frac{dy}{dx}$ = 1 + logx

$\Rightarrow \frac{dy}{dx} = x^x(1 + \log x)$

Since the function is decreasing,

$\Rightarrow x^x x(1 + |\log x) < 0$

$\Rightarrow 1 + \log x < 0$

$\Rightarrow \log x < -1$

$\Rightarrow x < \frac{1}{e}$

Therefore, function is decreasing on (0,$\frac{1}{e}$)

51.

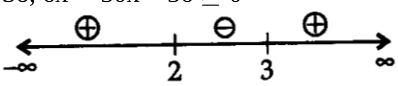
**(b)** $(-\infty$, 2] $\cup$ [3, $\infty$)

**Explanation:** Given, f(x) = $2x^3 - 15x^2 + 36x + 6$

$\therefore$ f'(x) = $6x^2 - 30x + 36$

It f'(x) $\geq$ 0, then f(x) is increasing.

So, $6x^2 - 30x + 36 \geq 0$




or, $x^2 - 5x + 36 > 0$

or, (x - 3)(x - 2) > 0

$\therefore$ x $\in$ $(-\infty$, 2] $\cup$ [3, $\infty$)

52.

**(c)** $\lambda > 1/2$

**Explanation:** $\lambda > 1/2$

53.

**(c)** a > 1

**Explanation:** a > 1

54. **(a)** local minima at x = 1

**Explanation:** Given, f(x) = $x^3 - 3x$

f'(x) = $3x^2 - 3$

For point of inflexion we have f'(x) = 0

$f'(x) = 0 \Rightarrow 3x^2 - 3 = 0 = 3(x-1)(x+1) \Rightarrow x = \pm 1$

Hence, f(x) has a point of inflexion at x = 0.

When , x is slightly less than 1, f'(x) = (+)(-)(+) i.e, negative

When x is slightly greater than 1, f'(x)= (+)(+)(+) i.e, positive

Hence, f'(x) changes its sign from negative to positive as x increases through 1 and hence x = 1 is a point of local minimum.

55.

**(b)** -1

**Explanation:** f(x) = $2x^3 - 3x^2 - 12x + 5$

$\Rightarrow$ f'(x) = $6x^2 - 6x - 12$

For local maxima or minima we have

f'(x) = 0

$6x^2$ - 6x - 12 = 0

$\Rightarrow x^2$ - x - 2 = 0

$\Rightarrow$ x = 2 or x = -1

f''(x) = 12x - 6

f''(2) = 18 > 0

function has local minima at x = 2.

f''(-1) = -18 < 0

function has local maxima at x = -1.

56.

**(b)** $\left(0, \frac{3}{2}\right) \cup (3, \infty)$

**Explanation:** Given,f(x) = $[x(x - 3)]^2$

$\Rightarrow$f'(x) = 2 [x(x - 3)][x$\frac{d(x-3)}{dx}$ + (x-3)$\frac{d(x)}{dx}$ ]

= f'(x) = 2 [x(x - 3)][x + x - 3]

= f'(x) = 2 [x(x - 3)][2x - 3]

= f'(x) = 0

$\Rightarrow$ x = 0, $x = \frac{3}{2}$ and x = 3

For increasing function f'(x) > 0

$\Rightarrow \left(0, \frac{3}{2}\right) \cup (3, \infty)$ function is increasing.

57.

**(d)** odd and increasing

**Explanation:** odd and increasing

58. **(a)** 4 + $\sqrt{2}$

**Explanation:** Maximum value of 4 $\sin^2$x + 3 $\cos^2$x = 4 sin 2x + 3(1 − $\sin^2$x) = $\sin^2$x + 3 is 4 as $0 \leq \sin^2 x \leq 1$ and that of sin $\frac{\pi}{2}$ + cos$\frac{\pi}{2}$ is $\frac{\pi}{\sqrt{2}} + \frac{\pi}{\sqrt{2}} = \sqrt{2}$ (as $-a^2 + b^2 \leq$ a sin x + b cos x $\leq a^2 + b^2$)

both attained at x = $\frac{\pi}{2}$

Hence, the given function has maximum value 4 + $\sqrt{2}$.

59. **(a)** $k \geq 1$

**Explanation:** Given,f(x) = sin x - kx

f'(x) = cos x - k

$\therefore$ f decreases, if f'$(x) \leq 0$

$\Rightarrow \cos x - k \leq 0$

$\Rightarrow \cos x \leq k$

Therefore, for decreasing $k \geq 1$

60.

**(b)** 5840

**Explanation:** 5840

61.

**(c)** decreasing in $\left(\frac{\pi}{2}, \pi\right)$

**Explanation:** We have, f(x) = 4 $\sin^3$x - 6 $\sin^2$x + 12 sin x + 100

$\therefore$ f'(x) = 12 $\sin^2$ x $\cdot$ cos x - 12 sin x $\cdot$ cos x + 12 cos x

= 12 cos x [$\sin^2$x - sin x + 1]

= 12 cos x [$\sin^2$ x + (1 - sinx)]

Now 1 - sin x $\geq$ 0 and $\sin^2$ x $\geq$ 0

$\therefore$ $\sin^2$ x + 1 - sin x > 0

Hence f'(x) > 0, when cos x > 0 i.e., $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

So, f(x) is increasing when $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

and f'(x) < 0, when cos x < 0 i.e., $x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

Hence, f(x) is decreasing when $x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

Hence, f(x) is decreasing in $\left(\frac{\pi}{2}, \pi\right)$

62.

**(b)** -1 < k < 1

**Explanation:** -1 < k < 1

63. **(a)** k > 3

**Explanation:** f(x) = $kx^3 - 9x^2 + 9x + 3$

f'(x) = $3kx^2 - 18x + 9$

= $3(kx^2 - 6x + 3)$

Given: f(x) is monotonically increasing in every interval.

⇒ f'(x) > 0

⇒ $3(kx^2 - 6x + 3) > 0$

⇒ $(kx^2 - 6x = 3) > 0$

⇒ K > 0 and $(-6)^2 - 4(k)(3) < 0$ [∵ $ax^2 + bx + c > 0$ and D is c < 0]

⇒ k > 0 and $(-6)^2 - 4(k)(3) < 0$

⇒ k > 0 and 36 - 12k < 0

⇒ k > 0 and 12k > 36

⇒ k > 0 and k > 3

⇒ k > 3

64. **(a)** $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

**Explanation:** $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$Given function,f(x) is sin x

∴ f'(x) = cos x

⇒ f'(x) = cos x

= 0

⇒ for $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

f'(x) is increasing,we

∴ f(x) is increasing in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.

Which is the required solution.

65.

**(d)** $e^{1/e}$

**Explanation:** Let y = f(x) = $\frac{1}{x}^x$

Then, log y = log$\frac{1}{x}^x$ = xlog$\frac{1}{x}$ = -xlogx

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = -\left(x.\frac{1}{x} + logx.1\right) = -(1 + \log x)$

$\Rightarrow f'(x) = -\frac{1}{x}^x (1 + \log x)$

$f'(x) = 0 \Rightarrow (1 + \log x) = 0$

$\Rightarrow \log x = -1 \Rightarrow x = \frac{1}{e}$

The maximum value of f(x) = $= f\left(\frac{1}{e}\right) = e^{1/e}$

66.

**(c)** $\frac{1}{4}$

**Explanation:** Given f(x) = $x^{25}(1 - x)^{75}$

f'(x) = $x^{25}. 75(1 - x)^{74}(-1)+(1 - x)^{75}. 25x^{24}$

= $25x^{24}(1 - x)^{74}\{-3x + (1-x)\}$

= $25x^{24}(1 - x)^{74}(1 - 4x)$

For maximum value of f(x) we have f'(x) = 0

⇒ $25x^{24}(1 - x)^{74}(1 - 4x) = 0$

⇒ x = 0, x = 1, x= $\frac{1}{4}$

All the values of x ∈ [0,1]

Note that f(0) = f(1) = 0 and f($\frac{1}{4}$) = $\frac{3^{75}}{4^{100}}$

So, f(x) is maximum at x = $\frac{1}{4}$

67.

**(c)** (-∞, 0)

**Explanation:** Given, f(x) = $x^2$

∵ f'(x) = 2x < 0 ∀ x ∈(-∞, 0)

∴ f is strictly decreasing function in (-∞, 0).

68. **(a)** $x = \frac{-\pi}{2}$

**Explanation:** We can go through options for this question

Option a is wrong because 0 is not included in $(-\pi, 0)$

At $x = \frac{-\pi}{4}$ value of f(x) is $-\sqrt{2}$ = -1.41

At $x = \frac{-\pi}{3}$ value of f(x) is -2.

At $x = \frac{-\pi}{2}$ value of f(x) = -1.

∴ f(x) has max value at $x = \frac{-\pi}{2}$

Which is -1.This is the required solution.

69.

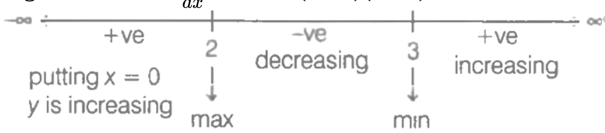
**(d)** 39, 38

**Explanation:** Let y = $2x^3 - 15x^2 + 36x + 11$ ...(i)

∴ $\frac{dy}{dx}$ = $6x^2 - 30x + 36 = 6(x^2 - 5x + 6)$

= 6(x - 2)(x - 3) ...(ii)

Sign scheme for $\frac{dy}{dx}$ i.e., for (x - 2)(x - 3) is




y has maximum value at x = 2

From Eq. (i), the corresponding maximum value of

y = $2(2)^3 - 15(2)^2 + 36(2) + 11 = 39$

y has minimum value at x = 3

From Eq. (i), minimum value of

y = $2(3)^3 - 15(3)^2 + 36(3) + 11 = 38$

70.

**(c)** $(-\infty, 1) \cup (2, 3)$

**Explanation:** Given that;

f(x) = 2 log(x - 2) - $x^2$ + 4x +1

f'(x) = $\frac{2}{(x-2)}$ - 2x + 4

= $\frac{2}{(x-2)}$ - 2(x - 2)

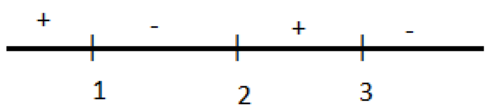
= $\frac{2(1-(x-2)^2)}{(x-2)}$

= $\frac{2(1-x+2)(1+x-2)}{(x-2)}$

= $\frac{2(3-x)(x-1)}{(x-2)}$

Critical points are;

1, 2 and 3




F(x) is increasing in $(-\infty, 1) \cup (2, 3)$

71.

**(c)** strictly increasing

**Explanation:** Let $x_1$ and $x_2$ be any two numbers in R,

where $x_1 < x_2$.

Given, $f(x) = e^{2x}$

Then, we have $x_1 < x_2 \Rightarrow 2x_1 < 2x_2$

$\Rightarrow e^{2x_1} < e^{2x_2}$

[$\because x_1 < x_2$ then $a^{x_1} < a^{x_2}$, when a > 1]

$f(x_1) < f(x_2)$.

Hence, f is strictly increasing on R.

72.

**(d)** $\frac{\pi}{2}$

**Explanation:** f(x) = x + cosx

f'(x) = 1 - sinx

For maximum and minimum values of f(x) , we have f'(x) = 0

Now, f'(x) = 0 $\Rightarrow 1 - sinx = 0 \Rightarrow x = \frac{\pi}{2}$

Hence, maximum value of f(x) is $f\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$

73. **(a)** -2

**Explanation:** Given,$f(x) = x^2 + kx + 1$

For increasing

f'(x) = 2x + k

$k \geq -2x$

thus,

$k \geq -2x$

Least value of -2

74.

**(b)** neither maximum value nor minimum value

**Explanation:** Given, $f(x) = x^3 + 1$

$\therefore f'(x) = 3x^2$ and f''(x) = 6x

Put f'(x) = 0

$\Rightarrow 3x^2 = 0 \Rightarrow x = 0$

At x = 0, f''(x) = 0

Thus, f(x) has neither maximum value nor minimum value.

75. **(a)** a > 0

**Explanation:** f(x) = ax

f'(x) = a

f(x) is increasing on $\frac{1}{2}$ if a > 0