Solution

CET25M8 APPLICATION OF INTEGRALS

Class 12 - Mathematics

1.

(d) $\frac{91}{30}$

Explanation: The area bounded by the curve y = x^4 - $2x^3$ + x^2 + 3
with x-axis and ordinates
Minimum value of y when x = 0 is y = 3
Minimum value of y when x = 1 is y = 3
$\Rightarrow \int_0^1 \left(x^4 - 2x^3 + x^2 + 3\right) dx$
$\Rightarrow \left[\frac{x^5}{5} - 2\frac{x^4}{4} + \frac{x^3}{3} + 3x\right]_0^1$
$\Rightarrow \frac{1}{5} - \frac{2}{4} + \frac{1}{3} + 3$
$\Rightarrow \frac{91}{30}$

2.

(d) $\frac{256}{3}$

Explanation: Since area = $2\int_0^{16} \sqrt{y} dy$, solve the integral to compute the value.

3.

(b) $\frac{8}{3}$

Explanation: The tangents are
$y = mx + \frac{a}{m}, y = mx + \frac{1}{2m},$ since $a = \frac{1}{2}$.
It passes through (-2, 0).
$\therefore 4m^2 = 1 \Rightarrow m = \pm\frac{1}{2}$
The tangents are:
$y = \frac{x}{2} + 1, y = -\frac{x}{2} - 1$
Required area:
$= 2\int_0^3 \left(\frac{y^2}{2} - 2y + 2\right) dy$
$= 2\left[\frac{4}{3} - 4 + 4\right]$
$= \frac{8}{3}$ sq.units

4.

(c) $A_n + A_{n-2} = \frac{1}{n-1}$

Explanation: Give y = $(\tan x)^n$ and the lies x = 0, y = 0, x = $\frac{\pi}{4}$
$A_n = \int_0^{\frac{\pi}{4}} \tan^n x dx$
$A_n = \int_0^{\frac{\pi}{4}} \tan^{n-2} x \left(\tan^2 x\right) dx$
$A_n = \int_0^{\frac{\pi}{4}} \tan^{n-2} x \left(\sec^2 x - 1\right) dx$
$A_n = \int_0^{\frac{\pi}{4}} \left(\tan n - 2x \sec^2 x - \tan^{n-2} x\right) dx$
$A_n = \int_0^{\frac{\pi}{4}} \tan n - 2x \sec^2 x dx - \int_0^{\frac{\pi}{4}} \tan^{n-2} x dx$
$A_n = \int_0^{\frac{\pi}{4}} \tan^{n-2} x \sec^2 x dx - A_{n-2}$
$A_n + A_{n-2} = \int_0^{\frac{\pi}{4}} \tan^{n-2} x \sec^2 x dx$
Put tan x = t ⇒ $\sec^2$ xdx = dt

X	0	$\frac{\pi}{4}$
t	0	1

$A_n + A_{n-2} = \int_0^1 t^{n-2} dt$
$A_n + A_{n-2} = \left[\frac{t^{n-1}}{n-1}\right]_0^1$
$A_n + A_{n-2} = \frac{1}{n-1}$

5.

(c) 4

Explanation: $\int_0^{\frac{16}{m^2}} [4\sqrt{x} - mx]\, dx = \frac{2}{3}$
$\Rightarrow \left[\frac{2}{3}\left(4x^{\frac{3}{2}}\right) - \frac{mx^2}{2}\right]_0^{\frac{16}{m^2}} = \frac{2}{3}$
$\Rightarrow \frac{8}{3}\left(\frac{16}{m^2}\right)^{\frac{3}{2}} - \frac{m}{2}\left(\frac{16}{m^2}\right)^2 = \frac{2}{3}$
$\Rightarrow \frac{8}{3}\left(\frac{64}{m^3}\right) - \frac{128}{m^3} = \frac{2}{3}$
$\Rightarrow \frac{1}{m^3}\left(\frac{128}{3}\right) = \frac{2}{3}$
$\Rightarrow \frac{1}{m^3} = \frac{1}{64}$
$\Rightarrow m = 4$

6.

(d) $\frac{1}{6}$ sq. units

Explanation: Given slope of the curve is 2x + 1.
$\therefore \frac{dy}{dx} = 2x + 1 \Rightarrow y = x^2 + x + c$
Also , it passes through (1, 2).
$\therefore 2 = 1 + 1 + c \Rightarrow c = 0$

Equation of curve is: y =x^2 +x. Therefore , points of intersection of y = x (x +1) and the x – axis are x = 0 , x = - 1.
Required area:
$\int_{-1}^{0} (x^2 + x)dx$
$= \left[\frac{x^3}{3} + \frac{x^2}{2}\right]_{-1}^{0}$
$= \left|-\frac{1}{6}\right| = \frac{1}{6}$ sq. units

7. **(a)** 1

Explanation: The given curves are : (i) y = x – 1 , x > 1 . (ii) y = - (x – 1) , x < 1. (iii) y = 1 these three lines enclose a triangle whose area is : $\frac{1}{2}$.base.height = $\frac{1}{2}$.2 .1 = 1 sq. unit.

8.

(c) $\frac{2}{3}$

Explanation: The area of the region bounded by
the curve x^2 = 4y and line x = 2 and x-axis
$\Rightarrow \int_0^2 ydx = \int_0^2 \frac{x^2}{4}dx$
$\Rightarrow \int_0^2 ydx = \left[\frac{x^3}{12}\right]_0^2$
$\Rightarrow \int_0^2 ydx = \frac{8}{12} = \frac{2}{3}$

9.

(c) $\frac{\pi}{6} - \frac{\sqrt{3}-1}{8}$

Explanation: Consider given equations as

x^2 + y^2 - 6x - 4y + 12 = 0, y = x, x = $\frac{5}{2}$

$\Rightarrow$ intersection points $\left(\frac{5}{2}, \frac{5}{2}\right)$ and (2, 2)

x^2 + y^2 - 6x - 4y + 12 = 0

$\Rightarrow (x - 3)^2 + (y - 2)^2 = 1$

$\Rightarrow y = \sqrt{1-(x-3)^2} + 2$

$A = \int_2^{\frac{5}{2}} (x - \sqrt{1-(x-3)^2} + 2)dx$

$A = \frac{\pi}{6} - \frac{\sqrt{3}-1}{8}$

10.

(c) $\frac{16}{3}$

Explanation: Given parabola $y^2 = 8x$

$\Rightarrow 4a = 8$

$\Rightarrow a = 2$

Parabola bounded by x-axis and latus rectum

$\Rightarrow \int\limits_0^2 \sqrt{8x}dx = 2\sqrt{2}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^2 = \frac{16}{3}$ square units

11. **(a)** 1

Explanation: Required area is:

$\int\limits_1^3 ydx = \int\limits_1^3 |x-2|\,dx$

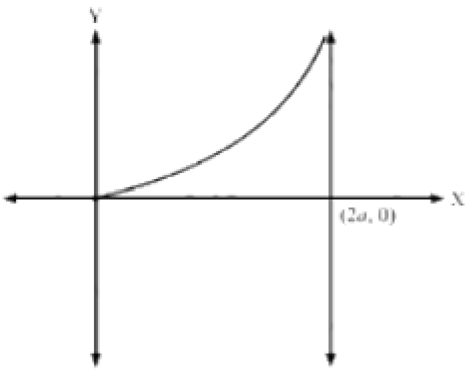
$= \int\limits_1^2 |x-2|\,dx + \int\limits_2^3 |x-2|\,dx$

$= \int\limits_1^2 |2-x|\,dx + \int\limits_2^3 |x-2|\,dx$

$= \left[2x - \frac{x^2}{2}\right]_1^2 + \left[\frac{x^2}{2} - 2x\right]_2^3 = 1$

12. **(a)** $\frac{3}{2}\pi a^2$

Explanation:



$y^2 (2a - x) = x^3$

$y = \sqrt{\frac{x^3}{2a-x}}$

Let x = $2a\sin^2\theta$

$\mathrm{dx} = 4\mathrm{a}\sin\theta\cos\theta\mathrm{d}\theta$

Area $= \int\limits_0^{2a} \sqrt{\frac{x^3}{2a-x}}dx$

$= \int\limits_0^{\frac{\pi}{2}} \sqrt{\frac{(8a^3)\sin^6\theta}{(2a)\cos^2\theta}} \cdot (4a)\sin\theta\cos\theta d\theta$

$= 8a^2\int\limits_0^{\frac{x}{2}} \sqrt{\sin^6\theta}\sin\theta d\theta$

$= 8a^2\left[\int\limits_0^{\frac{\pi}{2}} \sin^4\theta d\theta\right]$

$= 8a^2\left[\int\limits_0^{\frac{\pi}{2}} \sin^2\theta\left(1-\cos^2\theta\right)d\theta\right]$

$$= 8a^2\left[\int_0^{\frac{\pi}{2}}\left(\frac{1-\cos 2\theta}{2}\right)d\theta - \frac{1}{4}\int_0^{\frac{\pi}{2}}\sin^2 2\theta d\theta\right]$$
$$= 8a^2\left[\frac{1}{2}[\theta]_0^{\frac{\pi}{2}} - \left[\frac{\sin 2\theta}{4}\right]_0^{\frac{\pi}{2}}\right] - \frac{1}{4}\left[\int_0^{\frac{\pi}{2}}\frac{1-\cos 4\theta}{2}d\theta\right]$$
$$= 8a^2\left[\left(\frac{\pi}{4}\right) - 0\right] - \frac{1}{4}\left[\frac{\pi}{4} - 0\right]$$
$$= 8a^2\left[\frac{\pi}{4} - \frac{\pi}{16}\right] = \frac{3}{2}\pi a^2$$

13.

(c) 2π sq units

Explanation: Since Area = $4\int_0^{\sqrt{2}}\sqrt{2-x^2}$

= $4\left(\frac{x}{2}\sqrt{2-x^2} + \sin^{-1}\frac{x}{\sqrt{2}}\right)_0^{\sqrt{2}}$ = 2π sq. units

14.

(c) 9

Explanation: Required area:

$$\int_0^9\sqrt{x}dx - \int_3^9\left(\frac{x-3}{2}\right)dx = \left[\frac{x^{\frac{3}{2}}}{3/2}\right]_0^9 - \frac{1}{2}\left[\frac{x^2}{2} - 3x\right]_3^9 = 9 \text{ sq.units}$$

15. **(a)** $\frac{8}{3}$

Explanation: The two curves $y^2 = 4x$ and $y = x$ meet where $x^2 = 4x$ i.e ..where $x = 0$ or $x = 4$. Moreover, the parabola lies above the line $y = x$ between $x = 0$ and $x = 4$. Hence, the required area is:

$$\int_0^4\left(\sqrt{4x} - x\right)dx = \int_0^4\left(2x^{\frac{1}{2}} - x\right)dx$$
$$= \left[\frac{2x^{\frac{3}{2}}}{3/2} - \frac{x^2}{2}\right]_0^4$$
$$= \frac{4}{3}\left(4^{\frac{3}{2}}\right) - \frac{16}{2} = \frac{32}{3} - 8 = \frac{8}{3}$$

16. **(a)** $\frac{1}{6}$

Explanation: The area bounded by curve and coordinates axis :

$\int_0^1 ydx = \int_0^1(1-\sqrt{x})^2dx = \int_0^1(1-2\sqrt{x}+x)dx = 1 - \frac{4}{3} + \frac{1}{2} = \frac{1}{6}sq.\,units$ .Which is the required solution.

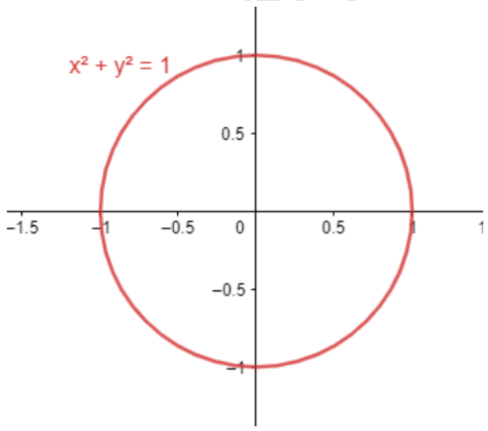
17.

(b) π sq units

Explanation:

Given;

The circle $x^2 + y^2 = 1$



By the symmetry of the circle with x-axis and y-axis.

Required area

= $4\int_0^1(\sqrt{1-x^2})dx$

$= \left[\int \sqrt{a^2 - x^2} dx = \frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]$

$= 4\int_0^1 (\sqrt{1^2 - x^2}) dx$

$= 4\left[\frac{x\sqrt{1^2-x^2}}{2} + \frac{1^2}{2}\sin^{-1}\left(\frac{x}{1}\right)\right]_0^1$

$= 4\left(0 - \frac{\pi}{4} - 0 - 0\right)$

$= \pi$ sq.units

18. **(a)** 1

Explanation: Required area:

$\int\limits_1^3 y dx = \int\limits_1^3 |x-2|\, dx$

$= \int\limits_1^2 |x-2|\, dx + \int\limits_2^3 |x-2|\, dx$

$= \int\limits_1^2 (2-x) dx + \int\limits_2^3 (x-2) dx$

$= \left[2x - \frac{x^2}{2}\right]_1^2 + \left[\frac{x^2}{2} - 2x\right]_2^3 = 1$

19. **(a)** $\frac{2}{3}$

Explanation: Required area :

$= 2\int\limits_0^a \sqrt{4ax} dx$

$= k\alpha(2\sqrt{4a\alpha})$

$= \frac{8\sqrt{a}}{3}\alpha^{\frac{3}{2}}$

$= 4\sqrt{a}k\alpha^{\frac{3}{2}} \Rightarrow k = \frac{2}{3}$

20.

(d) $\frac{4}{3}a^2$

Explanation: X - cordinate of latus rectum is a

$\Rightarrow \int_0^a y dx = \int_0^a 2\sqrt{a}\sqrt{x} dx$

$\Rightarrow \int_0^a y dx = 2\sqrt{a}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]$

$\Rightarrow \int_0^a y dx = \frac{4a^2}{3}$

21. **(a)** $\frac{1}{2}$ sq.units

Explanation: y = f(x) = x(x - 1)(x - 2) is +ve for x > 2, is – ve for 1 < x < 2; +ve for 0 < x < 1 , is – ve for x< 0.

Required area:

$\int\limits_0^1 y dx + \left|\int\limits_1^2 y dx\right|$

$\int\limits_0^1 (x^3 - 3x^2 + 2x) dx + \left|\int\limits_1^2 (x^3 - 3x^2 + 2x) dx\right|$

$= \left[\frac{x^4}{4} - x^3 + x^2\right]_0^1 + \left|\left[\frac{x^4}{4} - x^3 + x^2\right]_1^2\right|$

$= \frac{1}{2}$ sq. units

22.

(d) 4

Explanation: Reqd. area sq.units

$= \int_0^2 (y-2) dy + \int_2^0 (2-y) dy + \int_0^4 2 dy$

$= \left[\frac{y^2}{2} - 2y\right]_0^2 + \left[2y - \frac{y^2}{2}\right]_2^0 + [2y]_0^4$

= (2 - 4) - (4 - 2) + 8 = 4

23.

(b) $\frac{3}{4}(\pi - 2)$

Explanation: Required area:

= $\int_0^3 \left(\frac{1}{3}\sqrt{3^2-x^2}-\frac{1}{3}(3-x)\right)dx$

= $\frac{1}{3}\left[\frac{x\sqrt{3^2-x^2}}{2}+\frac{3^2}{2}\sin^{-1}\frac{x}{3}-3x+\frac{x^2}{2}\right]_0^3$

= $\frac{1}{3}\left[0+\frac{3^2}{2}\sin^{-1}1-3^2+\frac{3^2}{2}\right]$ -$\frac{1}{3}\left[0+\frac{3^2}{2}\sin^{-1}0-0+0\right]$

= $\frac{1}{3}\left[\frac{3^2}{2}.\frac{\pi}{2}-\frac{3^2}{2}-0\right]$

= $3/4\,(\pi-2)$

24.

(b) $\frac{9}{8}$ sq.units

Explanation: Eliminating y , we get :

$x^2-x-2=0 \Rightarrow x=-1,2$

Required area :

$=\int_{-1}^{2}\left(\frac{x}{4}+\frac{1}{2}-\frac{x^2}{4}\right)dx=\frac{1}{8}(4-1)+\frac{3}{2}-\frac{1}{12}(8+1)=\frac{3}{8}+\frac{3}{2}-\frac{3}{4}=\frac{9}{8}$ sq.units

25.

(c) $\frac{32}{3}$

Explanation: The area bounded by parabola x = 4 - y^2 and y-axis

As parabola bounded by y-axis x = 0

⇒ 4 - y^2 = 0

$\Rightarrow y=\pm 2$

$\int_{-2}^{2}\left(4-y^2\right)dy$

$\left[4y-\frac{y^3}{3}\right]_2^2$

$16-\left(\frac{8}{3}+\frac{8}{3}\right)$

= $\frac{32}{3}$

26.

(d) $\frac{9}{2}$ sq. units

Explanation: The equation y = 2x - x^2 i.e.y - 1 = - $(x-1)^2$ represents a downward parabola with vertex at (1,1) which meets x – axis where y = 0 .i .e . where x = 0, 2. Also , the line y = - x meets this parabola where – x = 2x - x^2 i.e. where x = 0 , 3.

Therefore, required area is:

$\int_0^3 (y_{parabola}-y_{line})dx=\int_0^3 (2x-x^2-(-x))dx=\left[\frac{3x^2}{2}-\frac{x^3}{3}\right]_0^3=\frac{27}{2}-9=\frac{9}{2}$ sq. units

27.

(c) 20π sq. units

Explanation: The area of the standard ellipse is given by ; πab. Here, a = 5 and b = 4 Therefore, the area of curve is $\pi(5)(4)=20\pi$.

28.

(c) $\frac{1}{6}$ sq. units

Explanation: Required area:

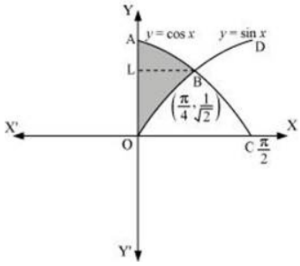
$\int_0^1 (x-x^2)dx$

$=\left[\frac{x^2}{2}-\frac{x^3}{3}\right]_0^1$

$=\frac{1}{6}$ sq.units.

29.

(b) $(\sqrt{2}-1)$ sq units

Explanation: 

It is given that the area bounded by the y-axis, y = cos x and y = sin x

⇒ sin x = cos x ⇒ $x=\frac{\pi}{4}$

Thus, the required area = Area ABLA + Area OBLO

$=\int_{\frac{1}{\sqrt{2}}}^{1} xdy+\int_{0}^{\frac{1}{\sqrt{2}}} xdy$

$=\int_{\frac{1}{\sqrt{2}}}^{1} \cos^{-1} ydy+\int_{0}^{\frac{1}{\sqrt{2}}} \sin^{-1} xdy$

$=\left[y\cos^{-1} y-\sqrt{1-y^2}\right]_{\frac{1}{\sqrt{2}}}^{1}+\left[x\sin^{-1} x+\sqrt{1-x^2}\right]_{0}^{\frac{1}{\sqrt{2}}}$

$\Rightarrow=\left[\cos^{-1}(1)-\frac{1}{\sqrt{2}}\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)+\sqrt{1-\frac{1}{2}}\right]+\left[\frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)+\sqrt{1-\frac{1}{2}}-1\right]$

$=\frac{-\pi}{4\sqrt{2}}+\frac{1}{\sqrt{2}}+\frac{\pi}{4\sqrt{2}}+\frac{1}{\sqrt{2}}-1$

$=\frac{2}{\sqrt{2}}-1$

$=\sqrt{2}-1$ units

30.

(d) $c^2\log\left(\frac{a}{b}\right)$

Explanation: Required area :

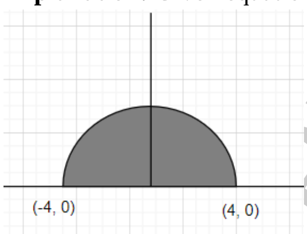
$=\int_{b}^{a}\frac{c^2}{x}dx=c^2[\log x]_b^a=c^2(\log a-\log b)=c^2log\left(\frac{a}{b}\right).$

Which is the required solution.

31.

(b) 8πsq units

Explanation: Given equation of curve is $y=\sqrt{16-x^2}$ and the equation of line is x - axis is



$\therefore\sqrt{16-x^2}=0$ .....(i)

⇒ 16 - x^2 = 0

⇒ x^2 = 16

⇒ x= ±4

So, the intersections points are (4, 0) and (-4, 0).

∴ Area of the curve, A = $\int_{-4}^{4}\left(16-x^2\right)^{12}dx$

= $\int_{-4}^{4}\sqrt{\left(4^2-x^2\right)}dx$

= $\left[\frac{x}{2}\sqrt{4^2-x^2}+\frac{4^2}{2}\sin^{-1}\frac{x}{4}\right]_{-4}^{4}$

= $\left[\frac{4}{2}\sqrt{4^2-4^2}+8\sin^{-1}\frac{4}{4}\right]$ - $\left[-\frac{4}{2}\sqrt{4^2-(-4)^2}+8\sin^{-1}\left(-\frac{4}{4}\right)\right]$

= $\left[2\cdot 0+8\cdot\frac{\pi}{2}-0+8\cdot\frac{\pi}{2}\right]$ = 8πsq. units

Which is the required solution.

32. **(a)** 2 sq. units

Explanation: $\int_0^\pi y dx = \int_0^\pi \sin x dx$

$\int_0^\pi y dx = -[\cos x]_0^\pi$

$\int_0^\pi y dx = -[-1-1]$

$\int_0^\pi y dx = 2$

33.

(d) None of these

Explanation: The area of the region bounded by the parabola

$(y - 2)^2 = x - 1$ the tangent to it at the point with the ordinates 3 and y-axis

y = 3 ⇒ x = 2

$(y - 2)^2 = x - 1$

slope of tangent at x = 2

$2(y-2)\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{2(y-2)}$

$\left|\frac{dy}{dx}\right|_{(2,3)} = \frac{1}{2}$

Equation of tangent

$y - 3 = \frac{1}{2}(x - 2)$

$y = \frac{x}{2} + 2$

$A = \int_0^3 \left((y-2)^2 + 1 - 2(y-2)\right) dy$

$A = \int_0^3 (y-3)^2 dy$

$A = \left[\frac{(y-3)^3}{3}\right]_0^3$

A = 9 sq units

34.

(c) πa^2

Explanation: Required area:

$=2\int_0^a a\sqrt{\frac{a-x}{x}}dx = 2a\int_0^a \sqrt{\frac{a-x}{x}}dx$

Put $x = a\sin^2\theta$, On differentiating, we get $dx = 2a \ \sin\theta \ \cos\theta \ d\theta$

For Limits, at $x = 0, \ \theta = 0$ and at $x = a, \ \theta = \frac{\pi}{2}$

$= 2a\int_0^{\frac{\pi}{2}} \sqrt{\frac{a - a\sin^2\theta}{a\sin^2\theta}} 2a \ \sin\theta \ \cos\theta \ d\theta$

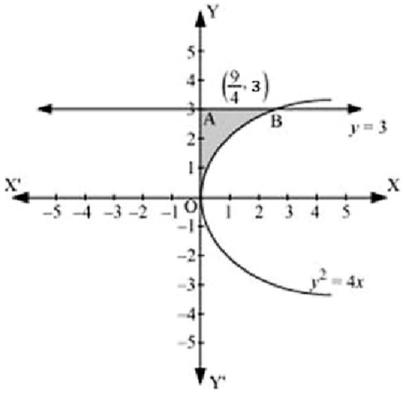
$= (2a)^2 \int_0^{\pi/2} \cos^2\theta d\theta$

$= 2a^2 . \int_0^{\pi/2} (1 + cos2\theta)d\theta$

$= 2a^2 . \left|\theta + \frac{\sin 2\theta}{2}\right|_0^{\pi/2}$

$= 2a^2 \left[\frac{\pi}{2} + 0 - 0\right] = \pi a^2$

35. **(a)** $\frac{9}{4}$

Explanation: 

The area bounded by the curve $y^2 = 4x$, y-axis and the line y = 3 is shown by shaded region in above figure.

Thus, Area OAB = $\int_0^3 x dy = \int_0^3 \frac{y^2}{4} dy$

$= \frac{1}{4}\left[\frac{y^3}{3}\right]_0^3$

$= \frac{1}{12}(27) = \frac{9}{4}$

Therefore, required area is = $\frac{9}{4}$ square units

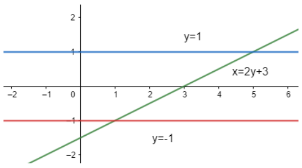
36.

(b) 6 sq units

Explanation:

Given;

The curve x = 2y + 3 and the y lines; y = 1 and y = –1



Required area

= $\int_{-1}^{1}(2y+3)dx$

= $\left[y^2+3y\right]_{-1}^{1}$

= (1 + 3 - 1 +3)

= 6 sq.units

37.

(b) 9

Explanation: To find area the curves y = $\sqrt{x}$ and x = 2y + 3 and x – axis in the first quadrant., We have ;

$y^2 - 2y - 3 = 0$,(y – 3) (y + 1) = 0 . y = 3 , - 1. In first quadrant , y = 3 and x = 9.

Therefore , required area is ;

$$\int_0^9 \sqrt{x}dx - \int_3^9 \left(\frac{x-3}{2}\right) = \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^9 - \frac{1}{2}\left[\frac{x^2}{2} - 3x\right]_3^9 = 9$$

38.

(d) 1

Explanation: Required area : $\left|\int_0^1 [(x-1)-(1-x)]dx\right| = 1$

39.

(b) $9\sec^{-1}(3) - \sqrt{8}$

Explanation: Required area:

$= 2\int_1^3 \sqrt{9-x^2}dx = 2\left[\frac{x\sqrt{9-x^2}}{2} + \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right)\right]_1^3$

$= 2\left[\frac{9}{2}\sin^{-1}(1) - \frac{2\sqrt{2}}{2} - \frac{9}{2}\sin^{-1}\left(\frac{1}{3}\right)\right]$

$= 9.\frac{\pi}{2} - 2\sqrt{2} - 9.\sin^{-1}\left(\frac{1}{3}\right)$
$= 9\left[\frac{\pi}{2} - \sin^{-1}\left(\frac{1}{3}\right)\right] - 2\sqrt{2}$
$= 9\cos^{-1}\frac{1}{3} - 2\sqrt{2}$
$= 9\sec^{-1}3 - \sqrt{8}$

40.

(c) 9 sq. units

Explanation: Given parabola is:
$(y-2)^2 = x - 1 \Rightarrow \frac{dy}{dx} = \frac{1}{2(y-2)}$
When y = 3, x = 2 $\therefore \frac{dy}{dx} = \frac{1}{2}$

Therefore, tangent at (2, 3) is y – 3 = $\frac{1}{2}$ (x – 2). i.e. x – 2y + 4 = 0, therefore required area is:
$\int_0^3 (y-2)^2 + 1.dy - \int_0^3 (2y-4)dy = \left[\frac{(y-2)^3}{3} + y\right]_0^3 - \left[y^2 - 4y\right]_0^3 = 9$ sq. units

41. **(a)** $\frac{\pi-2}{4}$ sq. units

Explanation: $x^2 + y^2 = 1, x + y = 1$
Meets when
$x^2 + (1-x)^2 = 1$
$\Rightarrow x^2 + 1 + x^2 - 2x = 1$
$\Rightarrow 2x^2 - 2x = 0 \Rightarrow 2x(x-1) = 0$

i.e. points (1 ,0) ,(0 ,1). Therefore , required area is ;
$\int_0^1 \left(\sqrt{1-x^2} - (1-x)\right) dx$
$= \left[\frac{x\sqrt{1-x^2}}{2} + \frac{1}{2}\sin^{-1}x - x + \frac{x^2}{2}\right]_0^1$

42. **(a)** $\frac{9}{2}$ sq. units

Explanation: The area bounded by y = 2 - x^2 and x + y = 0 ⇒ y = -x
2 - x^2 = -x
x^2 - x - 2 = 0
⇒ (x - 2) (x + 1) = 0
⇒ x = 2 or x = -1
$\int_{-1}^{2} \left(2 - x^2 - x\right) dx$
$\left[2x - \frac{x^3}{3} + \frac{x^2}{2}\right]_1^2$
$2(2+1) - \left(\frac{8}{3} + \frac{1}{3}\right) + \left(2 - \frac{1}{2}\right)$
$6 - 3 + \frac{3}{2}$
$\frac{9}{2}$ sq. units

43.

(c) $\frac{64}{3}$

Explanation: Required area: = $\int_{-6}^{2} \left[\frac{y^2}{4} - (3-y)\right] dy = \left[\frac{y^3}{12} - 3y + \frac{y^2}{2}\right]_{-6}^{2} = \frac{64}{3}$

44.

(d) 2 sq. units

Explanation: The angle bisectors of the line given by $x^2 - y^2 + 2y = 1$ are x = 0 , y = 1. Required area : = $\frac{1}{2}$ $.2.2 = 2\ sq.\ units$.

45.

(d) -2

Explanation: Required area:

$\int_0^{1-m} (x - x^2 - mx)dx$

$= \left[(1-m)\frac{x^2}{2} - \frac{x^3}{3}\right]_0^{1-m}$

$= \frac{(1-m)^3}{2} - \frac{(1-m)^3}{3} = \frac{9}{2}$

$\Rightarrow \frac{(1-m)^3}{6} = \frac{9}{2}$

$\Rightarrow$ m = - 2

46.

(c) $\frac{4(8\pi - \sqrt{3})}{3}$

Explanation: Required area :

$4\int_0^4 \sqrt{16 - x^2}dx$ - $\int_0^2 \sqrt{6x}dx + \int_2^4 \sqrt{(16 - x^2)}dx$ = $4\left[\frac{x\sqrt{16-x^2}}{2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_0^4 - 2\sqrt{6}\left[\frac{x^{3/2}}{3/2}\right]_0^2 - 2\left[\frac{x\sqrt{16-x^2}}{2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_0^4$ =

$\frac{4}{3}(8\pi - \sqrt{3})$

47.

(b) $\frac{1}{6} sq.\ units$

Explanation: Required area :

$\int_0^1 (\sqrt{x} - x)dx$

$= \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{x^2}{2}\right]_0^1$

$= \frac{1}{6} sq. units$

48.

(b) 2 : 3

Explanation: Let y^2 = 4x be a parabola and let x = b be a double ordinate. Then , A_1 = area enclosed by the parabola y^2 = 4ax and the double ordinate x = b.

$2\int_0^2 ydx = 2\int_0^b \sqrt{4ax}dx$

$= 4\sqrt{a}\int_0^b \sqrt{x^3}dx$

$= 4\sqrt{a}\left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^b$

$= 4\sqrt{a}.\frac{2}{3}b^{\frac{3}{2}} = \frac{8}{3}a^{\frac{1}{2}}b^{\frac{3}{2}}$ ....(1)

And,

A_2 = Area of the rectangle

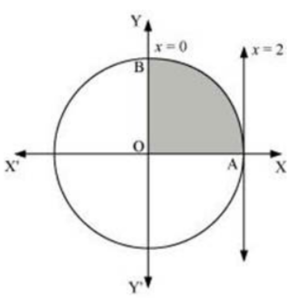
$= 2\sqrt{4ab}.b = 4a^{\frac{1}{2}}b^{\frac{3}{2}}$ ....(2)

Dividing (1) and (2),

$A_1 : A_2 = \frac{8}{3}a^{\frac{1}{2}}b^{\frac{3}{2}} : 4a^{\frac{1}{2}}b^{\frac{3}{2}} = 2 : 3$

49. **(a)** π

Explanation:



The area bounded by the circle and the lines, x = 0 and x = 2, in the first quadrant is shown by shaded region in above figure.

Area of OAB = $\int_0^2 ydx = \int_0^2 \sqrt{4-x^2}dx$

$= \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^{-2}$

$= 2\left(\frac{\pi}{2}\right) = \pi$

Therefore, required area is = π square units.

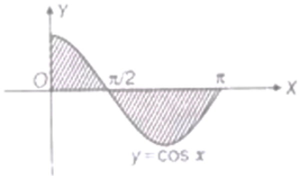
50. **(a)** $\frac{1}{2}$

Explanation: We have : $\int_{\frac{\pi}{3a}}^{\frac{\pi}{a}} \sin axdx = 3 \Rightarrow \left[-\frac{\cos ax}{a}\right]_{\frac{\pi}{3a}}^{\frac{\pi}{a}} = 3 \Rightarrow \frac{1}{a}\left(1+\frac{1}{2}\right) = 3 \Rightarrow a = \frac{1}{2}$

51. **(a)** 2 sq units

Explanation:

Required area enclosed by the curve y = cos x and x = π



A = $\int_0^{\pi/2} \cos xdx + \left|\int_{\pi/2}^{\pi} \cos xdx\right|$

= $\left[\sin\frac{\pi}{2} - \sin 0\right] + \left|\sin\frac{\pi}{2} - \sin\pi\right|$

= 1 + 1 = 2 sq. units

52.

(b) sin (3x + 4) + 3 (x -1) cos (3x + 4)

Explanation: Given that area bounded by the curve x-axis,

x = 1 and x = b

$\Rightarrow \int_1^b ydx = \int_1^b f(x)dx$

$\Rightarrow \int_1^b ydx - [A]_1^b$

$\Rightarrow \int_1^b f(x)dx = (b-1)\sin(3b+4)$

$\Rightarrow f(x) - \frac{d}{dx}[(x-1)\sin(3x+4)]$

$\Rightarrow$ 3(x - 1) cos (3x + 4) + sin (3x + 4)

53.

(b) $\frac{17}{4}$

Explanation: Required area

$\int_{-2}^{1} x^3dx$ = $\int_{-2}^{0} x^3dx$ + $\int_0^1 x^3dx$

= $\left[\frac{x^4}{4}\right]_{-2}^{0}$ + $\left[\frac{x^4}{4}\right]_0^1$

= $\left[0 - \frac{(-2)^4}{4}\right]$ + $\left[\frac{1}{4} - 0\right]$

= $\frac{16}{4}$ + $\frac{1}{4}$

= $\frac{17}{4}$

54.

(b) 4 sq. units

Explanation: Required area :

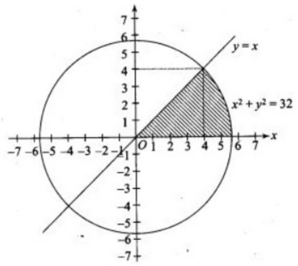
$= 2\int_0^{\pi} \sin xdx = 2[-\cos x]_0^{\pi} = 2[1+1] = 4 sq.units$

55.

(b) 4π sq units

Explanation:

We have, y = 0, y = x and the circle $x^2 + y^2 = 3$ in the first quadrant



Solving y = x with the circle

$x^2 + x^2 = 32$

$\Rightarrow x^2 = 16$

$\Rightarrow x = 4$

When x = 4, y = 4

For point of intersection of circle with the x-axis,

Put y = 0

$\therefore x^2 + 0 = 32$

$\Rightarrow x = \pm 4\sqrt{2}$

So, the circle intersects the x-axis at $(\pm 4\sqrt{2}, 0)$

From the figure, area of shaded region

A = $\int_0^4 x dx + \int_4^{4\sqrt{2}} \sqrt{(4\sqrt{2})^2 - x^2} dx$

= $\left[\frac{x^2}{2}\right]_0^4 + \left[\frac{x}{2}\sqrt{(4\sqrt{2})^2 - x^2} + \frac{(4\sqrt{2})^2}{2}\sin^{-1}\frac{x}{4\sqrt{2}}\right]_0^{4\sqrt{2}}$

= $\frac{16}{2} + \left[0 + 16\sin^{-1}1 - \frac{4}{2}\sqrt{(4\sqrt{2})^2 - 16} - 16\sin^{-1}\frac{4}{4\sqrt{2}}\right]$

= $8 + \left[16 \cdot \frac{\pi}{2} - 2 \cdot \sqrt{16} - 16 \cdot \frac{\pi}{4}\right]$

= $8 + [9\pi - 8 - 4\pi] = 4\pi$ sq. units

56.

(b) $\frac{4\pi - 3\sqrt{3}}{3}$

Explanation: Required area:

= $2\int_1^2 \sqrt{4 - x^2} dx$

= $2\left[\frac{x\sqrt{4-x^2}}{2} + \frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_1^2$

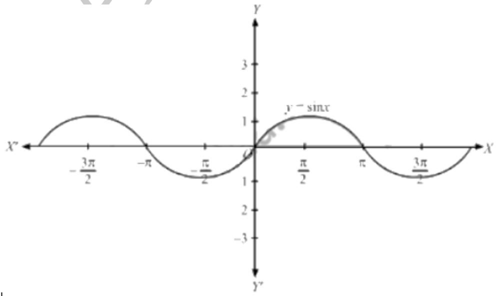
= $2\left[\frac{4}{2}\sin^{-1}(1) - \frac{\sqrt{3}}{2} - \frac{4}{2}\sin^{-1}\left(\frac{1}{2}\right)\right]$

= $\frac{4\pi - 3\sqrt{3}}{3}$

57.

(c) 1 sq units

Explanation:



The required area is given by the following equation ,

A = $\int_0^{\frac{\pi}{2}} y dx$

= $\int_0^{\frac{\pi}{2}} \sin(x) dx$

$= [-\cos(x)]_0^{\frac{\pi}{2}}$

= - cos ($\frac{\pi}{2}$) + cos 0

= 0 + 1 = 1 sq units

58.

(b) πab

Explanation: Area of standard ellipse is given by :πab.

59. **(a)** 1

Explanation: $\int_0^2 ydx = \frac{3}{\log_e 2}$

$\int_0^2 2^{kx}dx = \frac{3}{\log_e 2}$

$\left[\frac{2^{kx}}{\log_e 2}\right]_0^2 = \frac{3}{\log_e 2}$

2^{2k} - 1 = 3

2^{2k} = 4

$2^{2k} = 2^2$

⇒ 2k = 2

⇒ k = 1

60.

(b) $\frac{9}{2}$

Explanation: The two curves parabola and the line meet where,

3 - x = x^2 + 1 ⇔ x^2 + x - 2 = 0 ⇔ x = - 2,1

Therefore, the required area is:

$$\int_{-2}^{1}(y_{line} - y_{parabola})dx$$

$$= \int_{-2}^{1}\left\{3 - x - (x^2 + 1)\right\}dx$$

$$= \left[2x - \frac{x^2}{2} - \frac{x^3}{3}\right]_{-2}^{1} = \frac{9}{2}$$

61.

(b) 4

Explanation: Given that y = cos x, $0 \leq x \leq 2\pi$

$$\Rightarrow \int_0^{2\pi} ydx = \int_0^{\frac{\pi}{2}} ydx - \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} ydx + \int_{\frac{3\pi}{2}}^{2\pi} ydx$$

$$\Rightarrow \int_0^{2\pi} ydx = \int_0^{\frac{\pi}{2}} \cos x dx - \int_{\frac{1}{2}}^{\frac{3\pi}{2}} \cos x dx + \int_{\frac{3\pi}{2}}^{2\pi} \cos x dx$$

$$\Rightarrow \int_0^{2\pi} ydx = [\sin x]_0^{\frac{\pi}{2}} - [\sin x]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} + [\sin x]_{\frac{3\pi}{2}}^{2\pi}$$

$$\Rightarrow \int_0^{2\pi} ydx = 1 - 0 - (-1 - 1) + (0 + 1)$$

$$\Rightarrow \int_0^{2\pi} ydx = 4 \text{ sq. units}$$

62.

(b) 9

Explanation: The two curves meet where;

$\sqrt{x} = \frac{x-3}{2}$ ...(i)

⇒ 4x = x^2 - 6x + 9

⇒ x^2 - 10x + 9 = 0 ⇒ x = 9,1.

Therefore, the two curves meet where x = 9.

Therefore, required area:

$$= \int_0^9 \sqrt{x}dx - \int_3^9 \frac{x-3}{2}dx = \frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^9 - \frac{1}{2}\left[\frac{(x-3)^2}{2}\right]_3^9 = 9$$

63.

(b) $\frac{2}{3}$

Explanation: Required area :

$$= \left|\int_{-1}^{1} x\,|x|\,dx\right|$$

$$= \left|\int_{-1}^{0} x\,|x|\,dx + \int_{0}^{1} x\,|x|\,dx\right|$$

$$= \left|\int_{-1}^{0} -x^2dx\right| + \int_{0}^{1} x^2dx$$

$$= \left|\left[\frac{-x^3}{3}\right]_{-1}^{0}\right| - \left[\frac{x^3}{3}\right]_0^1$$

$= \frac{2}{3}$ sq. units