Solution

CET25M9 DIFFERENTIAL EQUATIONS

Class 12 - Mathematics

1.

(b) y $e^x + x^2 = C$

Explanation: It is given that $e^x dy + (ye^x + 2x)\,dx = 0$

$\Rightarrow e^x \frac{dy}{dx} + ye^x + 2x = 0$

$\Rightarrow \frac{dy}{dx} + y = -2xe^{-x}$

This is equation in the form of $\frac{dy}{dx} + py = Q$ (where, p = 1 and Q = $-2xe^{-x}$)

Now, I.F = $e^{\int pdx} = e^{\int dx} = e^x$

Thus, the solution of the given differential equation is given by the relation:

$y(I.F.) = \int (Q \times I.F.)dx + C$

$\Rightarrow ye^x = \int \left(-2xe^{-x} \cdot e^x\right) dx + C$

$\Rightarrow ye^x = -\int 2xdx + C$

$\Rightarrow ye^x = -x^2 + C$

$\Rightarrow ye^x + x^2 = C$

2. **(a)** $(2, \frac{3}{2})$

Explanation: The pairs $\left(2, \frac{3}{2}\right)$ is not feasible. Because the degree of any differential equation cannot be rational type. If so, then we use rationalization and convert it into an integer.

3.

(b) not defined

Explanation: not defined

4.

(b) $cos\frac{y-1}{x} = a$

Explanation: $\frac{dy}{dx} = cos^{-1}a$

$\int dy = cos^{-1}a \int dx$

$y = xcos^{-1}a + c$

When y = 1 , x = 0 , then 1=0 $cos^{-1}a + c \quad c = 1$

$\therefore y = xcos^{-1}a + 1$

$\therefore \frac{y-1}{x} = cos^{-1}a$

5.

(c) $y(1 + x^2) = c + \tan^{-1} x$

Explanation: We have, $\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{1}{\left(1+x^2\right)^2}$

Which is linear differential equation.

Here, P = $\frac{2x}{1+x^2}$ and Q = $\frac{1}{\left(1+x^2\right)^2}$

$\therefore$ I.F. = $e^{\int \frac{2x}{1+x^2}dx} = e^{\log\left(1+x^2\right)} = 1 + x^2$

$\therefore$ the general solution is

$y\left(1+x^2\right) = \int \left(1+x^2\right) \frac{1}{\left(1+x^2\right)^2} + C$

$\Rightarrow y\left(1+x^2\right) = \int \frac{1}{1+x^2}dx + C$

$\Rightarrow y(1 + x^2) = \tan^{-1} x + C$

6.

(d) 2

Explanation: Given differential equation is

y = $x\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^{-1} \Rightarrow$ y = $x\frac{dy}{dx} + \frac{1}{(dy/dx)}$

$\Rightarrow$ y$\left(\frac{dp}{dx}\right)$ = x$\left(\frac{dy}{dx}\right)^2$ + 1

$\therefore$ Degree = Power of highest derivative = 2

7.

(c) $y - x + 2 = log\left(x^2(y+2)^2\right)$

Explanation: $\frac{ydy}{y+2} = \frac{(x+2)dx}{x}$

$\int \frac{ydy}{y+2} = \int \frac{(x+2)dx}{x}$

$\int \frac{y+2-2dy}{y+2} = \int \frac{(x+2)dx}{x}$

$\int dy - \int \frac{2}{y+2} = \int dx + \int \frac{2}{x}$

$y - 2log|y+2| = x + 2log|x| + c$

Here x=1 and y=-1 implies

$-1 - 2log|-1+2| = 1 + 2log|1| + c \implies -1 - 2log|1| = 1 + c \because log|1| = 0 \implies \therefore c = -2$

Hence,

$y - 2log|y+2| = x + 2log|x| - 2$

$y - x + 2 = 2log|x| + 2log|y+2|$

$y - x + 2 = 2log|x(y+2)|$

$y - x + 2 = log|x^2(y+2)^2|$

8.

(d) $\phi\left(\frac{y}{x}\right) = kx$

Explanation: We have,

$\frac{dy}{dx} = \frac{y}{x} + \frac{\phi\left(\frac{y}{x}\right)}{\phi'\left(\frac{y}{x}\right)}$ ....(i)

Put v = $\frac{y}{x}$

$\Rightarrow x\frac{dv}{dx} + v = \frac{dy}{dx}$

$\Rightarrow x\frac{dv}{dx} + v = v + \frac{\phi(v)}{\phi'(v)}$ .. from (i)

$\Rightarrow x\frac{dv}{dx} = \frac{\phi(v)}{\phi'(v)}$

$\Rightarrow \frac{\phi'(v)}{\phi(v)}\mathrm{d}v = \frac{\mathrm{d}x}{x}$

$\Rightarrow \int \frac{\phi'(v)}{\phi(v)}dv = \int \frac{dx}{x}$

$\Rightarrow$ log ϕ(v) = log |x|+log k

$\Rightarrow$ log $\phi\left(\frac{y}{x}\right)$ - log |x| = log k

$\Rightarrow \log\left[\frac{\phi\left(\frac{y}{x}\right)}{x}\right] = \log k$

$\Rightarrow \phi\left(\frac{y}{x}\right) = kx$

9.

(d) y = 2x - 4

Explanation: Let, $\frac{dy}{dx} = p$

$\therefore p^2 - xp + y = 0$

$y = xp - p^2$ .... (i)

$\Rightarrow \frac{dy}{dx} = (x-2p)\frac{dy}{dx} + p$

$\Rightarrow p = (x-2p)\frac{dp}{dx} + p$

$\therefore \frac{dp}{dx} = 0$

$\Rightarrow$ P is constant

from Eqn. (i), $y = x \cdot c - c^2$

$\therefore\ y = 2x - 4$ is the correct option

10.

(d) 2, 1

Explanation: We have $\left[1+\left(\frac{dy}{dx}\right)^2\right]=\frac{d^2y}{dx^2}$

$\therefore$ Order = 2 and degree = 1

11.

(c) $\frac{1}{x}$

Explanation: $\frac{1}{x}$

12.

(b) $x=\ \nu y$

Explanation: A homogeneous equation of the form $\frac{dy}{dx}=h\left(\frac{x}{y}\right)$ can be solved by making the substitution x= vy.so that it becomes variable separable form and integration is then possible

13. **(a)** not defined

Explanation: In general terms for a polynomial the degree is the highest power.

Degree of differential equation is defined as the highest integer power of highest order derivative in the equation

Here the differential equation is $\left(\frac{d^2y}{dx^2}\right)^2+\left(\frac{dy}{dx}\right)^2=x\sin\left(\frac{dy}{dx}\right)$

Now for degree to exist the given differential equation must be a polynomial in some differentials.

Here differentials mean $\frac{dy}{dx}$ or $\frac{d^2y}{dx^2}$ or .... $\frac{d^ny}{dx^n}$

The given differential equation is not polynomial because of the term $\sin\frac{dy}{dx}$ and hence degree of such a differential equation is not defined.

14. **(a)** Both (i) and (ii)

Explanation: Both (i) and (ii)

15.

(d) $\frac{ax^2}{2}+bx$

Explanation: $\frac{ax^2}{2}+bx$

16. **(a)** $y=\frac{1-x}{1+x}$

Explanation: $y=\frac{1-x}{1+x}$

17. **(a)** -1

Explanation: Given differential equation is

$(x^2 + x + 1)dy + (y^2 + y + 1)dx = 0$

$\Rightarrow (x^2 + x + 1)dy = -(y^2 + y + 1)dx$

$\Rightarrow \frac{dy}{(1+y+y^2)}=-\frac{dx}{(1+x+x^2)}$

$\Rightarrow \frac{dx}{(1+x+x^2)}+\frac{dy}{(1+y+y^2)}=0$

$\Rightarrow \int\frac{dx}{\left(x+\frac{1}{2}\right)^2+\frac{3^2}{4}}+\int\frac{dy}{\left(y+\frac{1}{2}\right)^2+\frac{3}{4}}=0$

$\Rightarrow \int\frac{dx}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}+\frac{dy}{\left(y+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}=0$ [on integrating]

$\Rightarrow \frac{1}{\left(\frac{\sqrt{3}}{2}\right)}\tan^{-1}\left\{\frac{\left(x+\frac{1}{2}\right)}{\frac{\sqrt{3}}{2}}\right\}+\frac{1}{\frac{\sqrt{3}}{2}}\tan^{-1}\left\{\frac{y+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right\}=\frac{2}{\sqrt{3}}\tan^{-1}C_1\ \left[\because\int\frac{dx}{a^2+x^2}=\frac{1}{a}\tan^{-1}\frac{x}{a}\right]$

$\Rightarrow \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right)=\frac{2}{\sqrt{3}}\tan^{-1}C_1$

$\Rightarrow \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+\tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right)=\tan^{-1}C_1$

$\Rightarrow \tan^{-1}\left\{\frac{\left(\frac{2x+1}{\sqrt{3}}\right)+\left(\frac{2y+1}{\sqrt{3}}\right)}{1-\left(\frac{2x+1}{\sqrt{3}}\right)\left(\frac{2y+1}{\sqrt{3}}\right)}\right\}=\tan^{-1}C1\ [\because \tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)]$

$\Rightarrow \frac{\sqrt{3}[(2x+1)+(2y+1)]}{3-(2x+1)\cdot(2y+1)}=C_1$

$\Rightarrow 2\sqrt{3}(x + y + 1) = 2C(1 - x - y - 2xy)$

$\Rightarrow (x + y + 1) = \frac{1}{\sqrt{3}}(1 - x - y - 2xy)$

On comparing with (x + y + 1) = A(1 + Bx + Cy + Dxy)

Here, A is parameter and B, C and D are constants.

The value of C = -1

18.

(b) $\log(1+y) = x - \frac{x^2}{2} + C$

Explanation: Here, $\frac{dy}{dx} = 1 - x + y - xy$

$\frac{dy}{dx} = 1 - x + y(1-x)$

$\frac{dy}{dx} = (1+y)(1-x)$

$\frac{\mathrm{dy}}{1+\mathrm{y}} = (1-\mathrm{x})\mathrm{dx}$

On integrating on both sides, we obtain

$\log(1+y) = x - \frac{x^2}{2} + c$

19.

(d) sin y = e^x (log x) + C

Explanation: Given x cos y dy = (xe^x log x + e^x)dx

$\cos y dy = \frac{(xe^x \log x + e^x)}{x} dx$

On integrating on both sides we obtain

$\sin y = \log_x \int e^x dx - \int \frac{1}{x}\left(\int e^x\right) dx + \int \frac{e^x}{x} dx$

$\sin y = \log x\, (e^x) - \int \frac{e^x}{x} dx + \int \frac{e^x}{x} dx + C$

sin y = e^x log x + C

20.

(b) 1, 1

Explanation: 1, 1

21.

(d) 0

Explanation: 0, because the particular solution is free from arbitrary constants.

22.

(b) -1

Explanation: Given differential equation is

$(x^2 + x + 1)dy + (y^2 + y + 1)dx = 0$

$\Rightarrow (x^2 + x + 1)dy = -(y^2 + y + 1)dx$

$\Rightarrow \frac{dy}{(1+y+y^2)} = -\frac{dx}{(1+x+x^2)}$

$\Rightarrow \frac{dx}{(1+x+x^2)} + \frac{dy}{(1+y+y^2)} = 0$

$\Rightarrow \int \frac{dx}{\left(x+\frac{1}{2}\right)^2 + \frac{3^2}{4}} + \int \frac{dy}{\left(y+\frac{1}{2}\right)^2 + \frac{3}{4}} = 0$

$\Rightarrow \int \frac{dx}{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} + \frac{dy}{\left(y+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = 0$ [on integrating]

$\Rightarrow \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} \tan^{-1}\left\{\frac{\left(x+\frac{1}{2}\right)}{\frac{\sqrt{3}}{2}}\right\} + \frac{1}{\frac{\sqrt{3}}{2}} \tan^{-1}\left\{\frac{y+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right\} = \frac{2}{\sqrt{3}} \tan^{-1} C_1 \left[\because \int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a}\right]$

$\Rightarrow \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right) = \frac{2}{\sqrt{3}} \tan^{-1} C_1$

$\Rightarrow \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + \tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right) = \tan^{-1} C_1$

$\Rightarrow \tan^{-1}\left\{\frac{\left(\frac{2x+1}{\sqrt{3}}\right) + \left(\frac{2y+1}{\sqrt{3}}\right)}{1 - \left(\frac{2x+1}{\sqrt{3}}\right)\left(\frac{2y+1}{\sqrt{3}}\right)}\right\} = \tan^{-1} C1$ [$\because \tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$]

$\Rightarrow \frac{\sqrt{3}[(2x+1)+(2y+1)]}{3-(2x+1)\cdot(2y+1)} = C_1$

$\Rightarrow 2\sqrt{3}(x + y + 1) = 2C(1 - x - y - 2xy)$

$\Rightarrow (x + y + 1) = \frac{1}{\sqrt{3}}(1 - x - y - 2xy)$

On comparing with (x + y + 1) = A(1 + Bx + Cy + Dxy)

Here, A is parameter and B, C and D are constants.
The value of B = -1

23.

(c) 3

Explanation: y = Ax + A^3
Let us find the differential equation representing it so we have to eliminate the constant A
Differentiate with respect to x
$\Rightarrow \frac{dy}{dx} = A$
Put back value of A in y
$\Rightarrow \mathrm{y} = \frac{\mathrm{dy}}{\mathrm{dx}}\mathrm{x} + \left(\frac{\mathrm{dy}}{\mathrm{dx}}\right)^3$
Now for the degree to exist the differential equation must be a polynomial in some differentials
Here differentials mean $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{d^2y}{dx^2}$ or .... $\frac{\mathrm{d^n y}}{\mathrm{dx^n}}$
The given differential equation is polynomial in differentials $\frac{\mathrm{d}y}{\mathrm{d}x}$
The degree of a differential equation is defined as the highest integer power of highest order derivative in the equation
The highest derivative is $\frac{\mathrm{d}y}{\mathrm{d}x}$ and highest power to it is 3
Hence degree is 3.

24.

(c) $y = 2\tan\frac{x}{2} - x + C$

Explanation: Given $\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$
$\frac{dy}{dx} = \frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}$
$\frac{dy}{dx} = \tan^2\frac{x}{2}$
$dy = dx\left(\tan^2\frac{x}{2}\right)$
on integrating on both sides, we
$y = 2\tan\frac{x}{2} - x + C$

25.

(c) (y - x) = C(1 + yx)

Explanation: Given $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$
$\frac{dy}{1+y^2} = \frac{dx}{1+x^2}$
On integrating on both sides, we obtain
$\tan^{-1}y = \tan^{-1}x + c$
$\tan^{-1}y - \tan^{-1}x = c$
$\frac{y-x}{1+yx} = c\left(\text{since } \tan^{-1}y - \tan^{-1}x = \frac{y-x}{1+yx}\right)$
y - x = C(1 + yx)

26.

(d) sin x - cos y

Explanation: sin x - cos y

27.

(d) y = log {k (y + 1) (e^x + 1)}

Explanation: Given differential equation
(e^x + 1)ydy = (y + 1)e^x dx
$\Rightarrow \frac{ydy}{y+1} = \frac{e^x}{e^x+1}dx$
$\Rightarrow \int\left(1 - \frac{1}{y+1}\right)dy = \int\frac{e^x}{e^x+1}dx$
⇒ y - log(y + 1) = log(e^x + 1) + log k
⇒ y = log (y + 1) + log (1 + e^x) + log k
⇒ y = log (k(1 + y)(1 + e^x))

28.

(b) $\frac{dy}{dx}+Py=Q$

Explanation: Here the degree and order of the equation is 1 and also is of the form $\frac{dy}{dx}+Py=Q$ hence it is linear differential equation in first order

29. **(a)** $\frac{1}{x^2}$

Explanation: $\frac{1}{x^2}$

30. **(a)** $y=(\tan x-1)+Ce^{-\tan x}$

Explanation: $\frac{dy}{dx}+\sec^2 x.y=\tan x.\sec^2 x \Rightarrow P=\sec^2 x, Q=\tan x.\sec^2 x$

$\Rightarrow I.F.=e^{\int \sec^2 x dx}=e^{\tan x}$

$\Rightarrow y.e^{\tan x}=\int \tan x \sec^2 x e^{\tan x} dx \Rightarrow y.e^{\tan x}=(\tan x-1)e^{\tan x}+C$

$\Rightarrow y=(\tan x-1)+Ce^{-\tan x}$

31. **(a)** $y^2 dx + (x^2 - xy - y^2)\ dy = 0$

Explanation: it is a homogeneous differential equation ,because the degree of each individual term is same i.e. 2.

32.

(c) y = $-e^{-x}$ + C

Explanation: Given differential equation is

$\log\left(\frac{dy}{dx}\right)$ + x = 0 ⇒ $\log\left(\frac{dy}{dx}\right)$ = -x

⇒ $\frac{dy}{dx}$ = e^{-x} ⇒ $\int dy=\int e^{-x}\cdot dx$

On integrating both sides, we get y

y = $-e^{-x}$ + C

which is the required general solution.

33.

(b) $e^y = e^{x^2} + c$

Explanation: We have

$\frac{dy}{dx}=2xe^{x^2-y}=2xe^{x^2}\cdot e^{-y}$

$\Rightarrow e^y\frac{dy}{dx}=2xe^{x^2}$

$\Rightarrow \int e^y dy=2\int xe^{x^2}dx$

Put x^2 = t in R.H.S integral, we get

2xdx = dt

$\Rightarrow \int e^y dy=\int e^t dt$

⇒ $e^y = e^t + C$

⇒ $e^y = e^{x^2} + C$

34. **(a)** 2

Explanation: We have $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{1/2}=\frac{d^2y}{dx^2}$

$\Rightarrow \left[1+\left(\frac{dy}{dx}\right)^2\right]^3=\left(\frac{d^2y}{dx^2}\right)^2$

So, the degree of differential equation is 2.

35.

(c) y’ = h(x)g(y)

Explanation: y’ = h(x)g(y) since we can segregate functions of y with dy and x with dx.

$\frac{dy}{dx}=h(x)g(y)$ and $\frac{dy}{g(y)}=h(x)dx$

36.

(b) $y=\frac{1}{x}-\cot x+\frac{C}{x\sin x}$

Explanation: $\frac{dy}{dx}+(\frac{1}{x}+\cot x)y=1 \Rightarrow P=\left(\frac{1}{x}+\cot x\right), Q=1$

$\Rightarrow I.F.=e^{\int\left(\frac{1}{x}+\cot x\right)dx}=e^{\log x+\log\sin x}=e^{\log(x\sin x)}=xsinx$

$\Rightarrow y(x \sin x) = \int 1.x \sin x \Rightarrow xy \sin x = -x \cos x + \sin x + c$

$xysinx = -xcosx + sinx + c$

Dividing by xsinx, we get

$y = -cotx + \frac{1}{x} + \frac{c}{xsinx}$

It is a linear differential equation in y in the form of $\frac{dy}{dx} + Py = Q$ hence solution is $y.IF = \int IF \ \ Q(x)dx + c$

37.

(b) straight line passing through origin

Explanation: We have

xdy - ydx = 0

⇒ xdy = ydx

$\Rightarrow \frac{dy}{y} = \frac{dx}{x}$

On integrating both sides, we get

log y = log x + log C

⇒ log y = log Cx

⇒ y = Cx

This is a straight line passing through origin.

38.

(c) $2\sin^{-1} y = x\sqrt{1-x^2} + \sin^{-1} x + C$

Explanation: $2\sin^{-1} y = x\sqrt{1-x^2} + \sin^{-1} x + C$

39.

(d) $2y - x^3 = cx$

Explanation: We have,

$x\frac{dy}{dx} - y = x^2$

$\Rightarrow \frac{dy}{dx} - \frac{y}{x} = x$

Comparing with $\frac{dy}{dx}$ - Py = Q

$\Rightarrow P = \frac{-1}{x}, Q = x$

I.F. = $e^{\int Pdx} = e^{\int \frac{-1}{x}dx} = e^{-\log x} = \frac{1}{x}$

Multiplying $\frac{1}{x}$ on both sides,

$\frac{1}{x}\frac{dy}{dx} - \frac{y}{x^2} = 1$

$\frac{d}{dx}\frac{y}{x} = 1$

$\int \frac{d}{dx}\frac{y}{x} = \int xdx$

$\frac{y}{x} = \frac{x^2}{2} + c$

$2y = x^3 + cx$

$2y - x^3 - cx=0$

40.

(b) 1, 4

Explanation: Given, $y = cx + c^2 - 3c^{3/2} + 2$ ...(i)

On differentiating both sides w.r.t. x, we get

$\frac{dy}{dx}$ = C ...(ii)

From Eqs. (i) and (ii), we have

$y = \frac{dy}{dx} \times x + \left(\frac{dy}{dx}\right)^2 - 3\left(\frac{dy}{dx}\right)^{3/2} + 2$

$\Rightarrow y - x\frac{dy}{dx} - \left(\frac{dy}{dx}\right)^2 - 2 = -3\left(\frac{dy}{dx}\right)^{3/2}$

$\Rightarrow \left[y - x\left(\frac{dy}{dx}\right) - \left(\frac{dy}{dx}\right)^2 - 2\right]^2 = 9\left(\frac{dy}{dx}\right)^3$

Hence, order is 1 and degree is 4.

41. **(a)** Not defined

Explanation: It is given that equation is $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$

The given differential equation is not a polynomial equation in its derivative

Therefore, its degree is not defined.

42.

(d) not defined

Explanation: The given differential equation is not a polynomial equation in terms of its derivatives, so its degree is not defined.

43. **(a)** y = vx

Explanation: y = vx

44.

(c) Only (i)

Explanation: Given differential equation is

y = 2 cos x + 3 sin x ...(i)

Now, $\frac{dy}{dx}$ = = -2sin x + 3cos x

= -(2 cos x + 3 sin x) = -y [from Eq. (i)]

$\therefore \frac{d^2y}{dx^2}$ + y = 0

So, only Statement (i) is correct.

45.

(c) (x - C) e^{x+y} + 1 = 0

Explanation: We have,

$\frac{dy}{dx} + 1 = e^{x+y}$

$\Rightarrow \frac{dy}{dx} = e^{x+y} - 1$ ...(1)

Let x + y = v

$\Rightarrow 1 + \frac{dy}{dx} = \frac{dv}{dx}$

$\Rightarrow e^{V} = \frac{dv}{dx}$ ....from (i)

$\Rightarrow$ dx = e^{-v} dv

$\Rightarrow \int dx = \int e^{-v} dv$

$\Rightarrow$ x = -e-v + c

$\Rightarrow$ x - c = $-e^{-(x+y)}$

$\Rightarrow$ (x - c)e^{x+y} + 1 = 0

46.

(c) one

Explanation: one

47. **(a)** $\frac{-e^{-by}}{b} = \frac{e^{ax}}{a} + C$

Explanation: We have, $\log\left(\frac{dy}{dx}\right) = (ax + by)$

$\frac{dy}{dx} = e^{ax+by}$

$\frac{dy}{e^{by}} = e^{ax}dx$

On integrating on both sides, we obtain

$-\frac{e^{-by}}{b} = \frac{e^{ax}}{a} + c$

48.

(c) Both (i) and (ii)

Explanation:

i. We have, $\frac{dy}{dx}$ = f(x) + x $\Rightarrow$ dy = [f(x) + x]dx

On integrating both sides, we get

$\int dy = \int [f(x) + x]$ dx $\Rightarrow$ y = $\int f(x)$ dx + $\frac{x^2}{2}$ + C

Let g(x) = $\int f(x)$dx + $\frac{x^2}{2}$

Thus, general solution is of the form y = g (x) + C

ii. Consider the given differential equation $\left(\frac{dy}{dx}\right)^2$ = f(x)

Clearly, the highest order derivative occurring in the differential equation is $\frac{dy}{dx}$ and its highest power is 2.

iii. Also, given equation is polynomial in the derivative. So the degree of a differential equation is 2.

49.

(d) $y^2 - x^2 = 4$

Explanation: Given that $y\frac{dy}{dx} = x$

$ydy = xdx$

$\int ydy = \int xdx$

$\frac{y^2}{2} = \frac{x^2}{2} + c$

When x = 0 and y = 2, we get

$\frac{-2^2}{2} = \frac{0^2}{2} + c$

c = 2

$\frac{y^2}{2} = \frac{x^2}{2} + 2$

$y^2 - x^2 = 4$

50.

(d) 1

Explanation: Degree: It is the power of highest derivative in a differential equation

$\therefore$ Degree = 1

51.

(c) y = $C_1e^{C_2x} + C_3$

Explanation: We have ,

y1y3 = y_2^2

$\Rightarrow \frac{\frac{dy}{dx}}{\frac{d^2y}{dx^2}} = \frac{\frac{d^2y}{dx^2}}{\frac{d^3y}{dx^3}}$

$\Rightarrow \frac{\frac{d^2y}{dx^2}}{\frac{dy}{dx}} = \frac{\frac{d^3y}{dx^3}}{\frac{d^2y}{dx^2}}$

$\Rightarrow \frac{\frac{d^2y}{dx^2}}{\frac{dy}{dx}} = \int \frac{\frac{d^3y}{dx^3}}{\frac{d^2y}{dx^2}}$

$\Rightarrow \log \frac{dy}{dx} = \log \frac{d^2y}{dx^2} + \log C$

$\Rightarrow \mathrm{C}\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\mathrm{d}^2\mathrm{y}}{\mathrm{dx}^2}$

$\Rightarrow \int Cdx = \int \frac{\frac{d}{dx}\left(\frac{dy}{dx}\right)}{\frac{dy}{dx}}$

$\Rightarrow \mathrm{Cx} + \mathrm{C}_1 = \log \frac{\mathrm{d}y}{\mathrm{dx}}$

$\Rightarrow \frac{dy}{dx}$ = $e^{Cx + C_1}$

$\Rightarrow \int dy = \int e^{Cx+C}1dx$

$\Rightarrow y = C_4e^{C_5x} + C_3$

$\Rightarrow y = c_1e^{c_2x} + c_3$

52.

(d) 2, degree not defined

Explanation: 2, degree not defined

53.

(b) $y = xe^{-x}$

Explanation: We have, $\frac{dy}{dx} + y = e^{-x}$

This is a linear differential equation.

On comparing it with $\frac{dy}{dx} + Py = Q$ we get

P = 1, $Q = e^{-x}$

I.F. = $e^{\int Pdx} = e^{\int dx} = e^x$

So, the general solution is:

$y \cdot e^x = \int e^{-x} e^x dx + C$

$\Rightarrow y \cdot e^x = \int dx + C$

$\Rightarrow$ $y.e^x = x + C$ ....(i)

Given that when x = 0 and y = 0

$\Rightarrow$ 0 = 0 + C

$\Rightarrow$ C = 0

Eq. (i) becomes $y.e^x = x$

$\Rightarrow$ $y = xe^{-x}$

54.

(b) straight line passing through origin

Explanation: Given that,

x dy - y dx = 0

$\Rightarrow$ x dy = y dx

$\Rightarrow \frac{dy}{y} = \frac{dx}{x}$

On integrating both sides, we get

log y = log x + log C

$\Rightarrow$ log y = log Cx

$\Rightarrow$ y = Cx

which is a straight line passing through the origin.

55. **(a)** $x^2 = y$

Explanation: We have,

$\frac{dy}{dx} = \frac{2y}{x}$

$\Rightarrow \frac{dy}{2y} = \frac{dx}{x}$

$\Rightarrow \int \frac{dy}{2y} = \int \frac{dx}{x}$

$\Rightarrow \log|y| = 2\log|x| + \log c$

$\Rightarrow \log|y| = \log x^2 + \log c$

$\Rightarrow$ log y = $log\left(x^2 c\right)$

$\Rightarrow$ $y = cx^2$

Tangent passing through (1,1) , c = 1

$\Rightarrow$ $y = x^2$

56.

(c) $\tan^{-1}\left(\frac{y}{x}\right) = \log x + C$

Explanation: We have,

$\frac{dy}{dx} = \frac{x^2+xy+y^2}{x^2}$

$\frac{dy}{dx} = 1 + \frac{y}{x} + \frac{y^2}{x^2}$ ...(i)

Let y = vx

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$1 + v + v^2 = v + x\frac{dv}{dx}$ ....from(i)

$1 + v^2 = x\frac{dv}{dx}$

$\frac{dx}{x} = \frac{dv}{1+v^2}$

$\int \frac{dx}{x} = \int \frac{dv}{1+v^2}$

log |x| = $\tan^{-1}$x + c

57.

(d) 1

Explanation: Given differential equation is

$\left(\frac{dy}{dx}\right)^2 + \frac{dy}{dx}$ - $\sin^2$y = 0

The highest order derivative, present in the differential equation is $\left(\frac{dy}{dx}\right)$.

Therefore, its order is one.

58.

(d) 3, 2

Explanation: We have, $\left(\frac{d^3y}{dx^3}\right)^2 - 3\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^4 = y^4$

∴ Order = 3 and degree = 2

59.

(b) $2y - 1 = (\sin x - \cos x) e^x$

Explanation: $\frac{dy}{dx} = e^x \sin x$

$\int dy = \int e^x \sin x dx$

$y = \frac{1}{2}(\sin x - \cos x) e^x + C$

When x = y = 0 , we get

$0 = \frac{1}{2}(\sin 0 - \cos 0) e^0 + C$

$C = \frac{1}{2}$

Hence, $y = \frac{1}{2}(\sin x - \cos x) e^x + \frac{1}{2}$

$2y - 1 = (\sin x - \cos x) e^x$

60. **(a)** 2 and 4

Explanation: We have

$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{4}} = -x^{\frac{1}{5}}$

$\Rightarrow \left(\frac{dy}{dx}\right)^{\frac{1}{4}} = -\left(x^{\frac{1}{5}} + \frac{d^2y}{dx^2}\right)$

$\Rightarrow \frac{dy}{dx} = \left(x^{\frac{1}{5}} + \frac{d^2y}{dx^2}\right)^4$

∴ Order = 2, Degree = 4

61.

(b) $y = \frac{x^4+c}{4x^2}$

Explanation: Here,

Integrating factor, I.F = $e^{\int \frac{2}{x} dx} = e^{2 \log x} = e^{\log x^2} = x^2$

Therefore, the solution is y.x^2 = $\int x^2 \cdot x dx = \frac{x^4}{4} + k$,

i.e. $y = \frac{x^4+c}{4x^2}$

62. **(a)** 3

Explanation: 3

63.

(b) sec x

Explanation: Given that,

cos x$\frac{dy}{dx}$ + y sin x = 1

$\Rightarrow \frac{dy}{dx}$ + y tan x = sec x

Here, P = tan x and Q = sec x

IF = $e^{\int Pdx}$

$= e^{\int \tan x dx}$
$= e^{\ln \sec x}$
∴ IF = sec x

64.

(b) 3, 1
Explanation: 3, 1

65. **(a)** $x^2 + y^2 = C_1x$

Explanation: We have, $\frac{dy}{dx} = \frac{y^2-x^2}{2xy}$

Let y = vx
$\frac{dy}{dx} = v + x\frac{dv}{dx}$
$\frac{x^2v^2-x^2}{2vx^2} = v + x\frac{dv}{dx}$
$\frac{v^2-1}{2v} - v = x\frac{dv}{dx}$
$\frac{-v^2-1}{2v} = x\frac{dv}{dx}$
$\frac{dx}{x} + \frac{2vdv}{v^2+1} = 0$
On integrating on both sides, we obtain
$\log x + \log(v^2 + 1) = C$
$\log(x(v^2 + 1) = C$
$x\left(\frac{y^2}{x^2} + 1\right) = c$
$y^2 + x^2 = Cx$

66.

(b) x (y + cos x) = sin x + c
Explanation: We have, $\frac{dy}{dx} + \frac{1}{x}y$ = sin x
Which is linear differential equation.
Here, P = $\frac{1}{x}$ and Q = sin x
∴ I.F. = $e^{\int \frac{1}{x}dx} = e^{\log x} = x$
∴ The general solution is
$y \cdot x = \int x \cdot \sin x dx + C$
= - x cos x - $\int -\cos x dx$
= -x cos x + sin x
⇒ x(y + cos x) = sin x + C

67.

(c) 2
Explanation: It is given that equation is $2x^2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$
We can see that the highest order derivative present in the given differential equation is $\frac{d^2y}{dx^2}$
Thus, its order is two.

68. **(a)** sec x
Explanation: Given that $\frac{dy}{dx}$ + y tan x - sec x = 0
Here, P = tan x, Q = sec x
IF = $e^{\int Pdx} = e^{\int \tan x dx}$
$= e^{\log \sec x}$
= sec x

69.

(d) k < 0
Explanation: We have,
$\frac{dy}{dx} - ky = 0$
$\frac{dy}{dx}$ = ky

$\frac{dy}{dx}$ = kdx

$\int \frac{dy}{y} = k\int dx$

log |y| = kx + c

y(0) = 1 ⇒ x = 0, y = 1

⇒ c = 0

⇒ log |y| = kx

⇒ ekx = y

Given that $e^{k\infty} = 0$

as $e^{-\infty} = 0$

⇒ k < 0

70.

(c) 0

Explanation: 0

71.

(c) (1 + x^2) dy + (1 + y^2) dx = 0

Explanation: If y = f(x) is solution of a differential equation, then differentiating y = f(x) will give the same differential equation.

Let us find the differential equation by differentiating y with respect to x.

⇒ $\tan^{-1}x + \tan^{-1}y = c$

Differentiating with respect to x

⇒ $\frac{1}{1+x^2} + \frac{1}{1+y^2}\left(\frac{dy}{dx}\right) = 0$

⇒ $\frac{(1+y^2)dx+(1+x^2)dy}{(1+x^2)(1+y^2)dx} = 0$

⇒ (1 + y^2)dx + (1 + x^2)dy = 0

72.

(d) $x^2 - 1 = C\left(1 + y^2\right)$

Explanation: We have,

xdx + ydy = x^2y dy - y^2x dx

x dx + y^2x dx = x^2y dy - y dy

$x\left(1 + y^2\right)dx = y\left(x^2 - 1\right)dy$

$\frac{xdx}{x^2-1} = \frac{ydy}{1+y^2}$

$\int \frac{xdx}{x^2-1} = \int \frac{ydy}{1+y^2}$

$\frac{1}{2}\int \frac{2xdx}{x^2-1} = \frac{1}{2}\int \frac{2ydy}{1+y^2}$

$\frac{1}{2}\log\left(x^2 - 1\right) = \frac{1}{2}\log\left(1 + y^2\right) + \log c$

$\log\left(x^2 - 1\right) = \log\left(1 + y^2\right) + \log c$

x^2 - 1 = $\left(1 + y^2\right)c$

73.

(d) $\frac{1}{x}$

Explanation: $\frac{1}{x}$

74.

(c) 2

Explanation: 2

75. **(a)** y = $xsin^{-1}x$ + $\sqrt[2]{1 - x^2} + \ C$

Explanation: $dy = sin^{-1}xdx$

$\int dy = \int sin^{-1}xdx$

$y = sin^{-1}x\int dx - \int\int dx.\frac{\mathrm{d}}{\mathrm{dx}}sin^{-1}xdx + c$

$$y = x sin^{-1}x - \int \frac{x}{\sqrt{1-x^2}}dx$$
$$y = x sin^{-1}x - \frac{1}{2}\int \frac{2x}{\sqrt{1-x^2}}dx$$
$$y = x sin^{-1}x + \frac{1}{2}2\sqrt{1-x^2} + c$$
$$y = x sin^{-1}x + \sqrt{1-x^2} + c$$